20. In an epicyclic gear (Fig. 11.32), the wheel A fixed to S_1 has 30 teeth and rotates at 500 rpm. B gears with A and is fixed rigidly to C, both being free to rotate on S_2. The wheels B, C and D have 50, 70 and 90 teeth respectively. If D rotates at 80 rpm in a direction opposite to that of A, find the speed of the shaft S_2. (104.5 rpm in same direction)

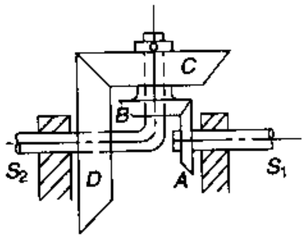


Fig. 11.32

21. In an indexing mechanism of a milling machine (Fig. 11.30), the drive is from gear wheels fixed to shafts S_1 and S_2 to the bevel A through the gear train. The number of teeth of A, B, D and F are 30, 60, 28 and 24 respectively. Each gear has a module of 10 mm.
Determine the number of revolutions of S (or A) for one revolution of S_1 when
(a) S_1 and S_2 have same speed in the same direction
(b) S_1 and S_2 have same speed in the opposite direction
(c) S_1 makes 48 rpm and S_2 is at rest
(d) S_1 makes 48 rpm and S_2 24 rpm in the same direction
(2; zero; 48 rpm; 72 rpm)

22. Show that in a Humpage reduction gear (Fig. 11.24), the wheel E rotates in the same direction as the wheel B if T_C/T_D is more than T_F/T_E and in the opposite direction if the same is less than T_F/T_E. Gear F is the fixed frame.

23. In a sun and planet gear train, the sun gear wheel having 60 teeth is fixed to the frame. Determine the numbers of teeth on the planet and the annulus wheels if the annulus rotates 130 times and the arm rotates 100 times, both in the same direction.
(70; 200)

24. A four-speed sliding gear box of an automobile is to be designed to give approximate speed ratios of 4, 2.4, 1.4 and 1 for the first, second, third and top gears respectively. The input and the output shafts have the same alignment. Horizontal central distance between them and the lay shaft is 98 mm. The teeth have a module of 4 mm. No wheel has less than 16 teeth. Calculate suitable number of teeth on each wheel and find the actual speed ratios attained.

25. In the pre-selective gear-box shown in Fig. 11.28, the number of teeth are
$T_{A1} = T_{A2} = 80$ $T_{S1} = T_{S2} = 24$
$T_{A3} = 68$ $T_{S3} = 21$
$T_{A4} = 90$ $T_{S4} = 41$
If the input shaft E rotates at a uniform speed of 640 rpm, determine the speeds of the output shaft F when different gears are engaged.
(147.7 rpm, 261.3 rpm, 423 rpm, 640 rpm, 101.4 rpm)

26. In the differential gear of a car shown in Fig. 11.30, the number of teeth on the pinion A on the propeller shaft is 24 whereas the crown gear B has 128 teeth. If the propeller shaft rotates at 800 rpm and the wheel attached to the shaft S_2 has a speed of 175 rpm, determine the speed of the wheel attached to shaft S_1 when the vehicle takes a turn.
(125 rpm)

1 ...C FORCE ...YSIS

Introduction

In all types of machinery, forces are transmitted from one component to the other such as from a belt to a pulley, from a brake drum to a brake shoe, from a gear to shaft. In the design of machine mechanisms, it is necessary to know the magnitudes as well as the directions of forces transmitted from the input to the output. The analysis helps in selecting proper sizes of the machine components to withstand the stresses developed in them. If proper sizes are not selected, the components may fail during the machine operations. On the other hand, if the members are designed to have more strength than required, the machine may not be able to compete with others due to more cost, weight, size, etc.

If the components of a machine accelerate, inertia forces are produced due to their masses. However, if the magnitudes of these forces are small compared to the externally applied loads, they can be neglected while analysing the mechanism. Such an analysis is known as *static-force analysis*. For example, in lifting cranes, the bucket load and the static weight loads may be quite high relative to any dynamic loads due to accelerating masses, and thus static-force analysis is justified.

When the inertia effect due to the mass of the components is also considered, it is called *dynamic-force analysis* which will be dealt in the next chapter.

CONSTRAINT AND APPLIED FORCES

A pair of action and reaction forces which constrain two connected bodies to behave in a particular manner depending upon the nature of connection are known as *constraint forces* whereas forces acting from outside on a system of bodies are called *applied forces*.

Constraint forces As the constraint forces at a mechanical contact occur in pairs, they have no net force effect on the system of bodies. However, for an individual body isolated from the system, only one of each pair of constraint forces has to be considered.

Applied forces Usually, these forces are applied through direct physical or mechanical contact. However, forces like electric, magnetic and gravitational are applied without actual physical contact.

STATIC EQUILIBRIUM

A body is in static equilibrium if it remains in its state of rest or motion. If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion. In static equilibrium

- the vector sum of all the forces acting on the body is zero, and
- the vector sum of all the moments about any arbitrary point is zero.

Mathematically,

$$\Sigma \mathbf{F} = 0 \quad (12.1)$$

$$\Sigma \mathbf{T} = 0 \quad (12.2)$$

In a planer system, forces can be described by two-dimensional vectors and, therefore,

$$\Sigma \mathbf{F}_x = 0 \quad (12.3)$$

$$\Sigma \mathbf{F}_y = 0 \quad (12.4)$$

$$\Sigma \mathbf{T}_z = 0 \quad (12.5)$$

EQUILIBRIUM OF TWO- AND THREE-FORCE MEMBERS

A member under the action of two forces will be in equilibrium if

- the forces are of the same magnitude,
- the forces act along the same line, and
- the forces are in opposite directions.

Figure 12.1 shows such a member.

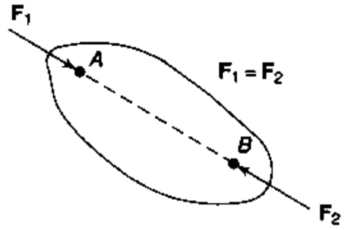


Fig. 12.1

A member under the action of three forces will be in equilibrium if

- the resultant of the forces is zero, and
- the lines of action of the forces intersect at a point (known as *point of concurrency*).

Figure 12.2 (a) shows a member acted upon by three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ and is in equilibrium as the lines of action of forces intersect at one point O and the resultant is zero. This is verified by adding the forces vectorially [Fig. 12.2 (b)]. As the head of the last vector $\mathbf{F}_3$ meets the tail of the first vector $\mathbf{F}_1$, the resultant is zero. It is not necessary to add the three vectors in order to obtain the resultant as is shown in Fig. 12.2 (c) in which $\mathbf{F}_2$ is added to $\mathbf{F}_3$ and then $\mathbf{F}_1$ is taken.

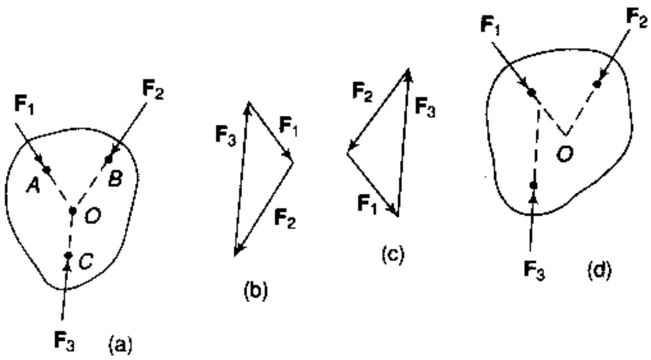


Fig. 12.2

Figure 12.2 (d) shows a case where the magnitudes and directions of the forces are the same as before, but the lines of action of the forces do not intersect at one point. Thus, the member is not in equilibrium.

Consider a member in equilibrium in which the force $\mathbf{F}_1$ is completely known, $\mathbf{F}_2$ is known in direction only and $\mathbf{F}_3$ is completely unknown. The point of applications of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are A, B and C respectively. To solve such a problem, first find the point of concurrency O from the two forces with known directions, i.e., from $\mathbf{F}_1$ and $\mathbf{F}_2$. Joining O with C gives the line of action of the third force $\mathbf{F}_3$. To know the magnitudes of the forces $\mathbf{F}_2$ and $\mathbf{F}_3$, take a vector of proper magnitude and direction to represent the force $\mathbf{F}_1$. From its two ends, draw lines parallel to the lines of action of the forces $\mathbf{F}_2$ and $\mathbf{F}_3$ forming a force triangle [Figs 12.2 (b) or (c). Mark arrowheads on $\mathbf{F}_2$ and $\mathbf{F}_3$ so that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in the same order.

If the lines of action of two forces are parallel then the point of concurrency lies at infinity and, therefore, the third force is also parallel to the first two.

12.4 MEMBER WITH TWO FORCES AND A TORQUE

A member under the action of two forces and an applied torque will be in equilibrium if

- the forces are equal in magnitude, parallel in direction and opposite in sense, and
- the forces form a couple which is equal and opposite to the applied torque.

Figure 12.3 shows a member acted upon by two equal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and an applied torque **T**. For equilibrium,

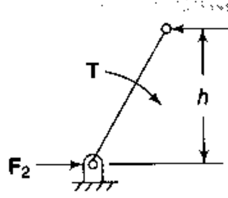


Fig. 12.3

$$T = F_1 \times h = F_2 \times h \tag{12.6}$$

where T, F_1 and F_2 are the magnitudes of **T**, $\mathbf{F}_1$ and $\mathbf{F}_2$ respectively. **T** is clockwise whereas the couple formed by $\mathbf{F}_1$ and $\mathbf{F}_2$ is counter-clockwise.

12.5 EQUILIBRIUM OF FOUR-FORCE MEMBERS

Normally, in most of the cases the above conditions for equilibrium of a member are found to be sufficient. However, in some problems, it may be found that the number of forces on a member is four or even more than that. In such cases, first look for the forces completely known and combine them into a single force representing the sum of the known forces. This may reduce the number of forces acting on a body to two or three. However, in planer mechanisms, a four-force system is also solvable if one force is known completely along with lines of action of the others. The following examples illustrate the procedure.

Example 12.1 *Figure 12.4(a) shows a quaternary link ABCD under the action of forces F_1, F_2, F_3 and F_4 acting at A, B, C and D respectively. The link is in static equilibrium. Determine the magnitude of the forces F_2 and F_3 and the direction of F_3.*

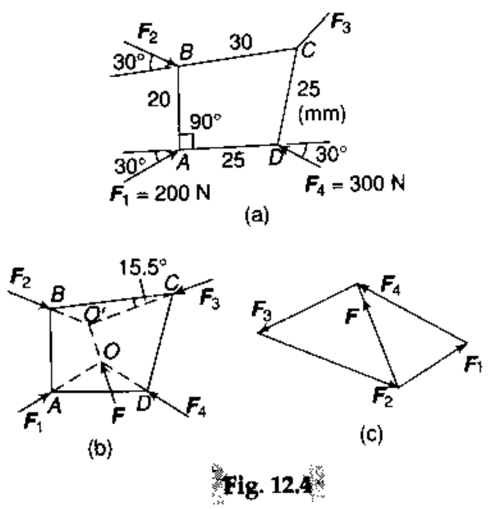


Fig. 12.4

Solution The forces $\mathbf{F}_1$ and $\mathbf{F}_4$ can be combined into a single force **F** by obtaining their resultant [Figs 12.4(b) and (c)]. The force **F** acts through O, the point where lines of action of $\mathbf{F}_1$ and $\mathbf{F}_4$ meet.

Now, the four-force member $ABCD$ is reduced to a three-force member under the action of forces **F** (completely known), $\mathbf{F}_2$ (only the direction known) and $\mathbf{F}_3$ (completely unknown).

Let **F** and $\mathbf{F}_2$ meet at O'. Then CO' is the line of action of force $\mathbf{F}_3$. By completing the force triangle, obtain the magnitude of $\mathbf{F}_2$ and $\mathbf{F}_3$.

Magnitude of $\mathbf{F}_2$ = 380 N

Magnitude of $\mathbf{F}_3$ = 284 N

Line of action of force $\mathbf{F}_3$ makes an angle of 15.5° with CB.

Example 12.2 *Figure 12.5(a) shows a cam with a reciprocating-roller follower system. Various forces acting on the follower are indicated in the figure. At the instant, an external force F_1 of 40 N, a spring force F_2 of*

15 N and cam force F_5 of unknown magnitude act on it along the lines of action as shown. F_3 and F_4 are the bearing reactions. Determine the magnitudes of the forces F_3, F_4 and F_5. Assume no friction.

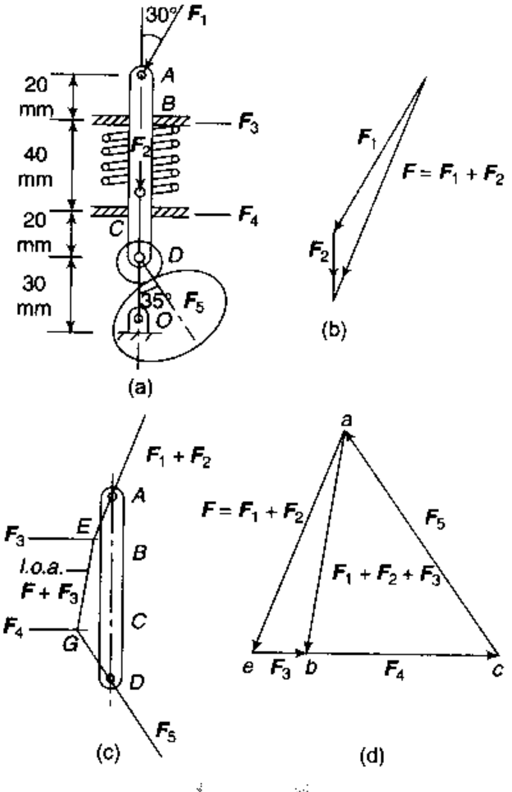


Fig. 12.5

Solution As in the previous example, forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be combined into a single force $\mathbf{F}$ by obtaining their resultant [Figs 12.5(b)]. Their resultant must pass through point A, the point of intersection of $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus, the number of forces acting on the body is reduced to four.

Now, assume that the magnitude of force $\mathbf{F}_3$ is known and the force $\mathbf{F}$ is to be combined with it. Then the resultant must pass through their point of intersection, i.e., the point E [Fig. 12.5(c)]. This way, the body becomes under the action of three forces which must be concurrent for the equilibrium of the body. Thus, the resultant of $\mathbf{F}$ and $\mathbf{F}_3$ must pass through the point G, the point of intersection of the forces $\mathbf{F}_4$ and $\mathbf{F}_5$. Therefore, the line of action of the resultant of $\mathbf{F}$ and $\mathbf{F}_3$ is EG.

Now since the force $\mathbf{F}$ is completely known and the lines of action of $\mathbf{F}_3$ and their resultant are known, the force diagram can be made. First take the force $\mathbf{F}$ and then to add $\mathbf{F}_3$ draw a line parallel to its line of action through the head of $\mathbf{F}$ [Fig. 12.5(d)]. Through the tail of vector $\mathbf{F}$ draw a line parallel to the line of action of the resultant. The triangle *aeb* thus provides the magnitude of the force $\mathbf{F}_3$ as well as resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

Now the number of forces acting on the body is reduced to three. One force is completely known and the lines of action of the other two are known. A triangle of forces can be drawn and magnitudes of $\mathbf{F}_3$, $\mathbf{F}_4$ and $\mathbf{F}_5$ can be found.

Magnitude of $\mathbf{F}_3$ = 12 N
Magnitude of $\mathbf{F}_4$ = 42 N
Magnitude of $\mathbf{F}_5$ = 60 N

12.6 FORCE CONVENTION

The force exerted by the member i on the member j is represented by $\mathbf{F}_{ij}$.

12.7 FREE-BODY DIAGRAMS

A free-body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.

Figure 12.6(a) shows a four-link mechanism. The free-body diagrams of its members 2, 3 and 4 are shown in Figs 12.6 (b) (c) and (d) respectively. Various forces acting on each member are also shown. As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.

Member 4 is acted upon by three forces $\mathbf{F}$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque $\mathbf{T}$.

Initially, the direction and the sense of some of the forces may not be known.

Assume that the force $\mathbf{F}$ on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known.

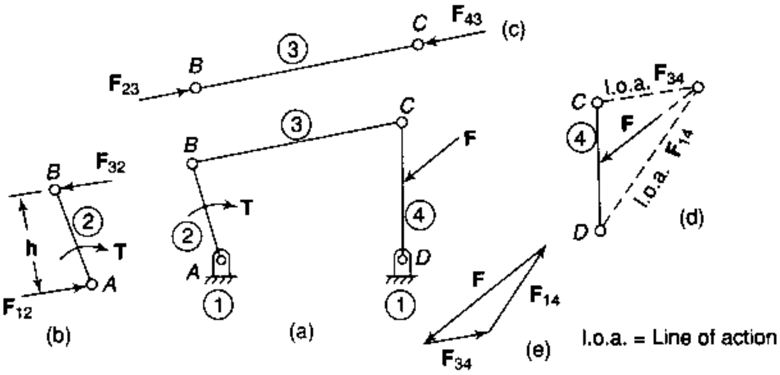


Fig. 12.6

Link 3 is a two-force member and for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along BC. Thus, $\mathbf{F}_{34}$, being equal and opposite to $\mathbf{F}_{43}$, also acts along BC. For the member 4 to be in equilibrium, $\mathbf{F}_{14}$ passes through the intersection of $\mathbf{F}$ and $\mathbf{F}_{34}$. By drawing a force triangle ($\mathbf{F}$ is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ can be known [Fig.12.6 (e)].

Now $$F_{34} = F_{43} = F_{23} = F_{32}$$

Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$$T = F_{12} \times h = F_{32} \times h$$

SUPERPOSITION

In linear systems, if a number of loads act on a system of forces, the net effect is equal to the superposition of the effects of the individual loads taken one at a time. A linear system is one in which the output force is directly proportional to the input force, i.e., in mechanisms where coulomb or dry friction is neglected.

Example 12.3 *A slider-crank mechanism with the following dimensions is acted upon by a force $F = 2$ kN at B as shown in Fig. 12.7(a):*

$OA = 100$ mm, $AB = 450$ mm.

Determine the input torque T on the link OA for the static equilibrium of the mechanism for the given configuration.

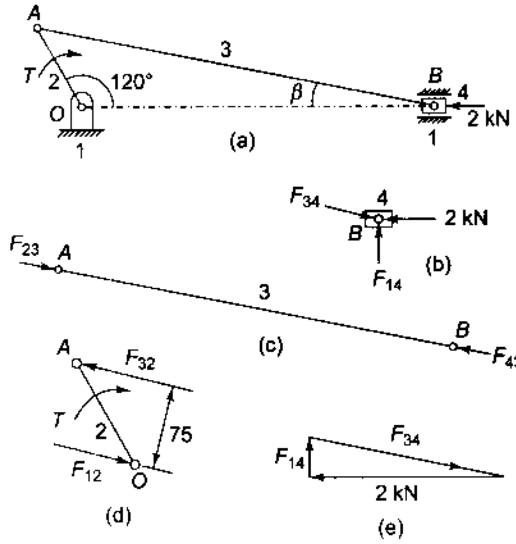


Fig. 12.7

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces $\mathbf{F}$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.7(b)].

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ [Fig. 12.7(c)].

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque $\mathbf{T}$ [Fig. 12.7(d)].

Initially, the direction and the sense of some of the forces are not known.

Now, adopt the following procedure:

- Force **F** on member 4 is known completely (= 2 kN, horizontal). To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, the link 3 will have to be considered first which is a two-force member.
- As the link 3 is a two-force member, for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along AB (at this stage, the sense of direction of forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ is not known). Thus, the line of action of $\mathbf{F}_{34}$ on member 4 is also along AB.
- As force $\mathbf{F}_{34}$ acts through the point B on the link 4, draw a line parallel to BC through B by taking a free body of the link 4 to represent the same. Now, since the link 4 is a three-force member, the third force $\mathbf{F}_{14}$ passes through the intersection of **F** and $\mathbf{F}_{34}$ [Fig. 12.7(b)]. By drawing a force triangle (**F** is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are known [Fig.12.7 (e)].

From force triangle,

$$F_{34} = 2.04 \text{ kN}$$

Now, $\quad F_{34} = -F_{43} = F_{23} = -F_{32}$

Member 2 will be in equilibrium [Fig. 12.7(e)] if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$T = F_{32} \times h = 2.04 \times 75 = -153$ kN.mm

(h = 75 mm on measurement)

The input torque has to be equal and opposite to this couple i.e.,

T = <u>153 kN.mm</u> or <u>153 N.m</u> (clockwise)

Analytical solution

$$\cos\beta = \frac{1}{n}\sqrt{n^2 - \sin^2\theta} = \frac{1}{4.5}\sqrt{4.5^2 - \sin^2 120°}$$

$= 0.981$

or $\beta = 11.1°$ (Refer Section 13.5)

$F_{34}\cos 11.1° = 2$ or $F_{34} = 2.04$ kN

$\angle OAB = 180° - 120° - 11.1° = 48.9°$

$\therefore T = F_{32} \times h = 2.04 \times 100 \sin 48.9°$

$= 153.7$ kN.mm

- The direction and senses of forces in the analytical solution can be known by drawing rough figures instead of drawing these to the scale.

Example 12.4 ***A four-link mechanism with the following dimensions is acted upon by a force 80 $\angle 150°$ N on the link DC [Fig. 12.8(a)]:***

AD = 500 mm, AB = 400 mm, BC = 1000 mm, DC = 750 mm, DE = 350 mm

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

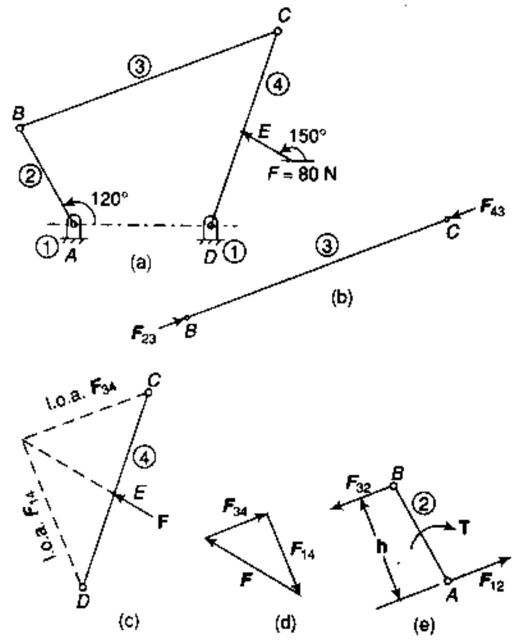


Fig. 12.8

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

Initially, the direction and the sense of some of the forces are not known.

Now, adopt the following procedure:

- Force **F** on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, the link 3 will have to be considered first which is a two-force member.

- As the link 3 is a two-force member (Fig. 12.8b), for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along BC (at this stage, the sense of direction of forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ is not known). Thus, the line of action of $\mathbf{F}_{34}$ is also along BC.
- As the force $\mathbf{F}_{34}$ acts through the point C on the link 4, draw a line parallel to BC through C by taking a free body of the link 4 to represent the same. Now, as the link 4 is a three-force member, the third force $\mathbf{F}_{14}$ passes through the intersection of $\mathbf{F}$ and $\mathbf{F}_{34}$ as the three forces are to be concurrent for equilibrium of the link [Fig. 12.8(c)]. By drawing a force triangle F is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are known [Fig.12.8 (d)].

From force triangle,

$F_{34} = 47.8$ N

Now, $F_{34} = -F_{43} = F_{23} = -F_{32}$

Member 2 will be in equilibrium [Fig. 12.8(e)] if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$T = -F_{32} \times h = -47.8 \times 393 = -18\ 780$ N.mm

The input torque has to be equal and opposite to this couple i.e.,

T = <u>18.78 N.m</u> (clockwise)

Analytical Method

First of all, the angular inclinations of the links BC and DC, i.e., angles β and ϕ are to be determined. This may be done by drawing the configuration or by analytical means (Section 4.1).

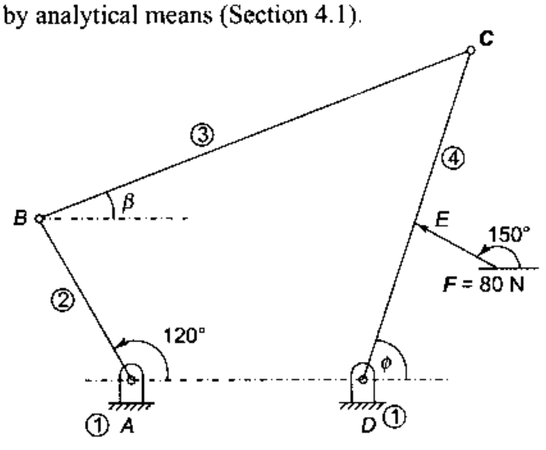


Fig. 12.9

We have (Fig. 12.9),

$2k = a^2 - b^2 + c^2 + d^2$

$k = (0.4^2 - 1^2 + 0.75^2 + 0.5^2)/2 = -0.01375$

$A = k - a(d - c)\cos\theta - cd = -0.01375 - 0.4(0.5 - 0.75)\cos 120° - 0.75 \times 0.5 = -0.439$

$B = -2ac\sin\theta = -2 \times 0.4 \times 0.75 \sin 120° = -0.52$

$C = k - a(d + c)\cos\theta + cd = -0.01375 - 0.4(0.5 + 0.75)\cos 120° + 0.75 \times 0.5 = 0.611$

$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \qquad \text{(Eq. 4.7)}$$

$$= 2\tan^{-1}\left[\frac{0.52 \pm \sqrt{(-0.52)^2 - 4\times(-0.439)(0.611)}}{2\times(-0.439)}\right]$$

$= 2\tan^{-1}(0.727 \text{ or } -0.439)$

$= 72° \text{ or } -47.4°$

Taking the first value (value in the first quadrant),

We have,

$a\sin\theta + b\sin\beta = c\sin\varphi$ (Eq. 4.3)

$0.4 \times \sin 120° + 1 \times \sin\beta = 0.75 \times \sin 72°$

or $\sin\beta = 0.712$ or $\beta = 21.5°$

Position vectors

$\mathbf{AB} = 0.4\angle 120°$, $\mathbf{BC} = 1.0\angle 21.5°$, $\mathbf{DC} = 0.75\angle 72°$, $\mathbf{DE} = 0.35\angle 72°$

The direction of $\mathbf{F}_{34}$ is along BC since it is a two-force member,

$\mathbf{F}_{34} = F_{34}\angle 21.5°$

As the link DC is in static equilibrium, no resultant forces or moments are acting on it.

Taking moments of the forces about point D,

$M_d = \mathbf{F}_4 \times \mathbf{DE} + \mathbf{F}_{34} \times \mathbf{DC} = \mathbf{0}$ (i)

Moments are the cross-multiplication of the vector, so it should be done in rectangular coordinates.

$\mathbf{F}_4 = 80\angle 150° = -69.28\,\mathbf{i} + 40\,\mathbf{j}$

$\mathbf{DE} = 0.35\angle 72° = 0.108\,\mathbf{i} + 0.333\,\mathbf{j}$

$\mathbf{F}_{34} = F_{34}\angle 21.5° = F_{34}(0.93\,\mathbf{i} + 0.367\,\mathbf{j})$

$\mathbf{DC} = 0.75\angle 72° = 0.232\,\mathbf{i} + 0.713\,\mathbf{j}$

Inserting the values of vectors in (i),

$(-69.28\,\mathbf{i} + 40\,\mathbf{j}) \times (0.108\,\mathbf{i} + 0.333\,\mathbf{j})$

$+ F_{34}(0.93\,\mathbf{i} + 0.367\,\mathbf{j}) \times (0.235\,\mathbf{i} + 0.712\,\mathbf{j}) = 0$

$$\text{or} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -69.28 & 40 & 0 \\ 0.108 & 0.333 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.93F_{34} & 0.367F_{34} & 0 \\ 0.232 & 0.713 & 0 \end{vmatrix} = 0$$

or $(-69.28 \times 0.333 - 40 \times 0.108) + (0.93\,F_{34} \times 0.713 - 0.367F_{34} \times 0.232)$

or $-27.4 + 0.58\,F_{34} = 0$ or $F_{34} = 47.3$ N

Thus, $\mathbf{F}_{34} = 47.3\angle 21.5°$

Now, $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 47.3\angle 21.5°$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = 47.3\angle 21.5°$

$T_{2c} = \mathbf{F}_{12} \times \mathbf{AB} = 47.3\angle 21.5° \times 0.4\angle 120° = 18.9$ N.m

Example 12.5 *A four-link mechanism with the following dimensions is acted upon by a force of 50 N on the link DC at the point E (Fig. 12.10a):*

AD = 300 mm, AB = 400 mm, BC = 600 mm, DC = 640 mm, DE = 840 mm

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

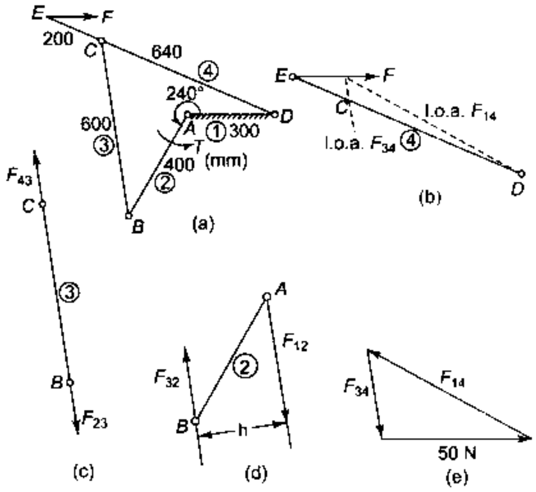


Fig. 12.10

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces $\mathbf{F}$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.10(b)]

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ [Fig. 12.10(c)]

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T** [Fig. 12.10(d)]

Initially, the direction and the sense of some of the forces are not known.

The procedure to solve the problem graphically is exactly similar to the previous example. In brief, the link 3 is a two-force member, so it provides the line of action of force $\mathbf{F}_{34}$ on the link 4. Since the link 4 is a three-force member and forces are to be concurrent, the lines of action of all the forces on the link 4 can be drawn. Then the force diagram provides the magnitude of various forces [Fig. 12.10(e)]. The rest of the procedure is self-explanatory.

From force triangle,

$F_{34} = 30.5$ N

Now, $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34}$ or $F_{32} = 30.5$ N

$T = F_{32} \times h = 30.5 \times 249 = 7595$ N.mm

($h = 249$ mm, on measurement)

The input torque has to be equal and opposite to the couple obtained by parallel forces i.e.,

$T = $ 7.595 N.m (counter clockwise)

Example 12.6 *For the mechanism shown in Fig. 12.11a, determine the torque on the link AB for the static equilibrium of the mechanism.*

Solution

(i) Composite Graphical Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

- Member 4 is acted upon by three forces $\mathbf{F}_1$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.11(b)].
- Member 3 is acted upon by three forces $\mathbf{F}_2$, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.
- Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

To solve the problem graphically, proceed as follows:

- Force $\mathbf{F}_1$ on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. However, as the link 3 now is a three-force member, it is not possible to know the direction of the force $\mathbf{F}_{34}$ from that also.

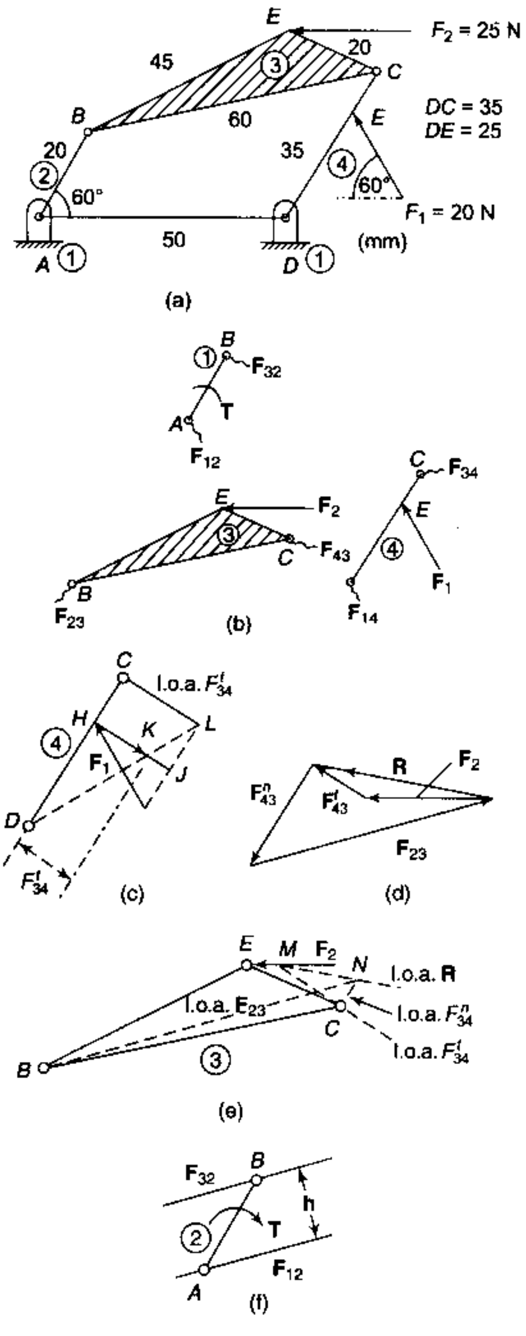


Fig. 12.11

- Consider two components, normal $\mathbf{F}_{34}^{n}$ and tangential $\mathbf{F}_{34}^{t}$ of the force $\mathbf{F}_{34}$. Assume $\mathbf{F}_{34}^{n}$ to be along *DC* and $\mathbf{F}_{34}^{t}$ perpendicular to *DC* through *C*. Also, take the components of force $\mathbf{F}_1$, i.e., $\mathbf{F}_1^{n}$ and $\mathbf{F}_1^{t}$ along the same directions.
- Now as the link 4 is in equilibrium, no moments are acting on it. Taking moments of all the forces acting on it about pivot point *D*.

  $M = \mathbf{F}_{34}^{t} \times DC + \mathbf{F}_1^{t} \times DE = 0$

  (No moments are to be there due to forces $\mathbf{F}_{34}^{n}$, $\mathbf{F}_1^{n}$ and $\mathbf{F}_{14}$ as these forces pass through the point *D*)

  or $\mathbf{F}_{34}^{t} = -\mathbf{F}_1^{t} \times \dfrac{DE}{DC}$

  Graphically, the above value of $\mathbf{F}_{34}^{t}$ can be obtained by taking $\mathbf{F}_1$ on the link 4 to some convenient scale and then taking two components of it, the normal component along *DC* and the tangential component perpendicular to *DC* being shown by *JH* in Fig. 12.11(c). Also, draw $CL \perp DC$. Draw *JL* parallel to *HC*. Join *DL* which intersects *JH* at *K*. Now, *KH* is the component $\mathbf{F}_{34}^{t}$ the direction being towards *K*.
- Now consider the equilibrium of the link 3. The forces acting on it are $\mathbf{F}_2$, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}^{t}$ and $\mathbf{F}_{43}^{n}$. The latter two components are equal and opposite to $\mathbf{F}_{34}^{t}$ and $\mathbf{F}_{34}^{n}$ respectively.
- Find the resultant of $\mathbf{F}_2$ and $\mathbf{F}_{43}^{t}$ by drawing the force diagram as shown in [Fig. 12.9(d)].
- Draw a line $CM \perp DC$ and through *C* to represent the line of action of force $\mathbf{F}_{43}^{t}$ on the link 3 [Fig. 12.11(d)]. It intersects the line of action of the force $\mathbf{F}_2$ at *M*. Now the resultant of $\mathbf{F}_2$ and $\mathbf{F}_{43}^{t}$ must pass through *M*. Thus, draw a line parallel to **R** through *M*.

  Now the link 3 is reduced to a three-force member [Fig. 12.11(e)], the forces being: **R**, $\mathbf{F}_{43}^{n}$ and $\mathbf{F}_{23}$.

  As these are to be concurrent forces, $\mathbf{F}_{23}$ must pass through the intersection of lines of forces $\mathbf{F}_{43}^{n}$ and **R**. Draw a line parallel to *DC* and through *C* to represent the line of action of force $\mathbf{F}_{43}^{n}$. This intersects the line of action of **R** at *N*. Join *BN*. Now *BN* represents the line of action of force $\mathbf{F}_{23}$.

- Complete the force diagram and find the magnitude of $\mathbf{F}_{23}$ and $\mathbf{F}^n_{43}$.
- Draw line parallel to line BN through B on link 2 [Fig. 12.11(f)] to represent the line of action of force $\mathbf{F}_{32}$ and a parallel line through A to represent the line of action of force $\mathbf{F}_{12}$.

  From force diagram,

  $F_{23} = 49.4$ N

  Now, $F_{32} = -F_{23} = -49.4$

  Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

  $T = -F_{32} \times h = -49.8 \times 14.3 = -706.4$ N.mm

  The input torque has to be equal and opposite to this couple, i.e.,

  $T = 706.4$ N.mm (clockwise)

(ii) Graphical Solution by Superposition method

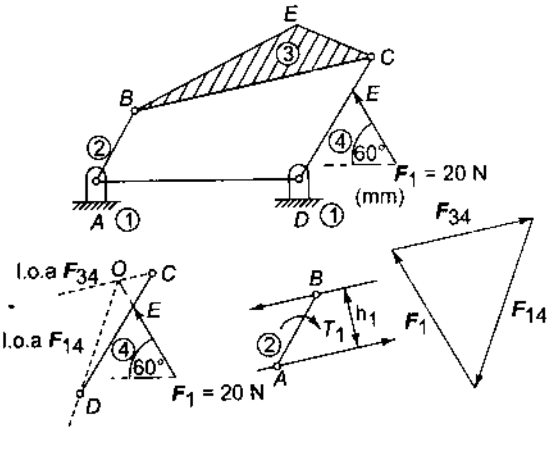


Fig. 12.12

Subproblem a (Fig. 12.12) Neglecting force $\mathbf{F}_2$

Link 4 is a three-force member in which only one force $\mathbf{F}_1$ is known. However, the line of action of $\mathbf{F}_{34}$ can be obtained from the equilibrium of the link 3 which is a two-force member and is acted upon by forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Thus, lines of action of forces $\mathbf{F}_{43}$ or $\mathbf{F}_{34}$ are along BC. If $\mathbf{F}_1$ and $\mathbf{F}_{34}$ intersect at O then line of action of $\mathbf{F}_{14}$ will be along OD since the three forces are to be concurrent. Draw the force triangle ($\mathbf{F}_1$ is completely known) and obtain the magnitudes of forces $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

$F_{34} = 17.6$ N

Also, $F_{34} = -F_{43} = F_{23} = -F_{32} = -17.6$ N

So, the direction of F_{32} is opposite to that of F_{23}.

Link 2 is subjected to two forces and a torque $\mathbf{T}_1$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_1 = F_{32} \times h_1 = 17.6 \times 14.9 = 262$ N.mm clockwise

Subproblem b (Fig. 12.13) Neglecting force $\mathbf{F}_1$.

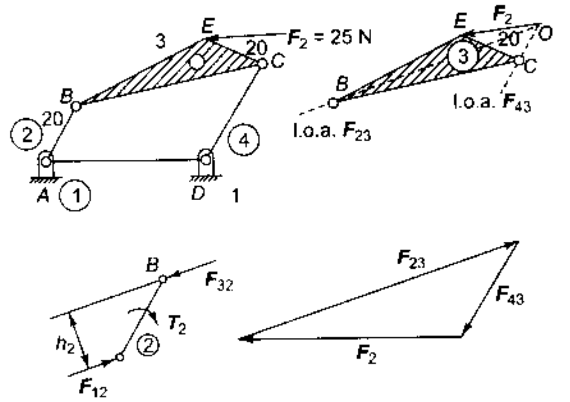


Fig. 12.13

Link 4 is a two-force member. The two forces $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are to be equal and opposite and their line of action is to be the same which shows that the line of action is along DC. Thus, the line of action of $\mathbf{F}_{43}$ is also along DC.

Link 3 is a three-force member in which $\mathbf{F}_2$ is completely known, only the direction of $\mathbf{F}_{43}$ is known (parallel to DC) and $\mathbf{F}_{23}$ is completely unknown. If the line of action of $\mathbf{F}_2$ and $\mathbf{F}_{43}$ meet at O, the line of action of $\mathbf{F}_{23}$ will be along OB as the three forces are to be concurrent. Draw the force triangle ($\mathbf{F}_2$ is completely known) by taking $\mathbf{F}_2$ to a suitable scale and two lines parallel to lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Mark arrowheads on $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ to know the directions.

$F_{23} = 33.2$ N

and $F_{23} = -F_{32} = -33.2$ N

So, direction of F_{32} is opposite to that of F_{23}.

Link 2 is subjected to two forces and a torque $\mathbf{T}_2$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_2 = F_{32} \times h_2 = 33.2 \times 13.2 = 438$ N.mm lockwise

Total torque = 262 + 438 = 700 N.mm

Example 12.7 *For the static equilibrium of the mechanism of [Fig. 12.14(a)], find the torque to be applied on link AB.*

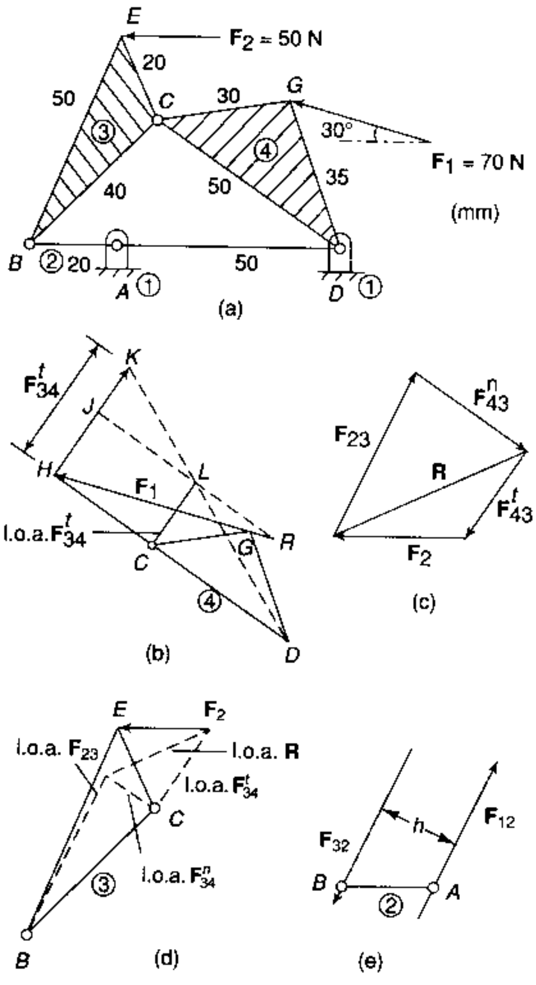


Fig. 12.14

Solution The point of action of force $\mathbf{F}_1$ on the link 4 is an offset point G. If DC is extended and let the line of action of force $\mathbf{F}_1$ meet at H then the force $\mathbf{F}_1$ may be considered to be acting on a virtual point H on the link DC as the magnitude of force as well as the magnitude couple effect is not going to vary.

Now, the problem can be solved by adopting the procedure given in the previous example. In brief:

- Take vector RH to represent force $\mathbf{F}_1$ to some scale.
- Find force $\mathbf{F}_{34}^t$. Its magnitude is given by HK and it acts through C.
- Find the resultant of $\mathbf{F}_2$ and $\mathbf{F}_{43}^t$ and its point of application in the free body diagram.
- Through point C, draw line for the vector $\mathbf{F}_{43}^n$ and then find the line of application of $\mathbf{F}_{23}$.

  From force diagram,

  $$F_{23} = 68.9 \text{ N}$$

  Now, $F_{32} = -F_{23} = -49.4$

  Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

  $$T = -F_{32} \times h = -68.9 \times 18.65 = -1285 \text{ N.mm}$$

  The input torque has to be equal and opposite to this couple, i.e.,

  $$T = 1.285 \text{ N.m (clockwise)}$$

- The example can also be worked out by the graphical method using the principle of superposition.

Example 12.8 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.15a, determine the input torque T_2 to be applied on the link AB for a force of 300 N on the slider D. The dimensions of the various links are*

OA = 400 mm, AB = 200 mm, OC = 800 mm, CD = 300 mm

Solution The slider at D or the link 6 is a three-force member. Lines of action of the forces are [Fig.12.15(b)]

- $\mathbf{F}$, 300 N as given
- $\mathbf{F}_{56}$ along CD, as link 5 is a two force member
- $\mathbf{F}_{16}$, normal reaction, perpendicular to slider motion

  Draw the force diagram and determine the direction sense of forces $\mathbf{F}_{56}$ and $\mathbf{F}_{16}$. From the force $\mathbf{F}_{56}$, the directions of forces $\mathbf{F}_{65}$, $\mathbf{F}_{35}$ and $\mathbf{F}_{53}$ are known. Now, the link 3 is a three-force member. Lines of action of the forces are
- $\mathbf{F}_{53}$, known completely through C

- $\mathbf{F}_{43}$, perpendicular to slider motion through B
- $\mathbf{F}_{13}$, unknown through A.

  As the lines of action of forces acting through B and C are known, the line of action of $\mathbf{F}_{13}$ through A must also pass through the point of intersection of the other two forces. Find the sense of the direction of force $\mathbf{F}_{43}$ by drawing the force triangle.

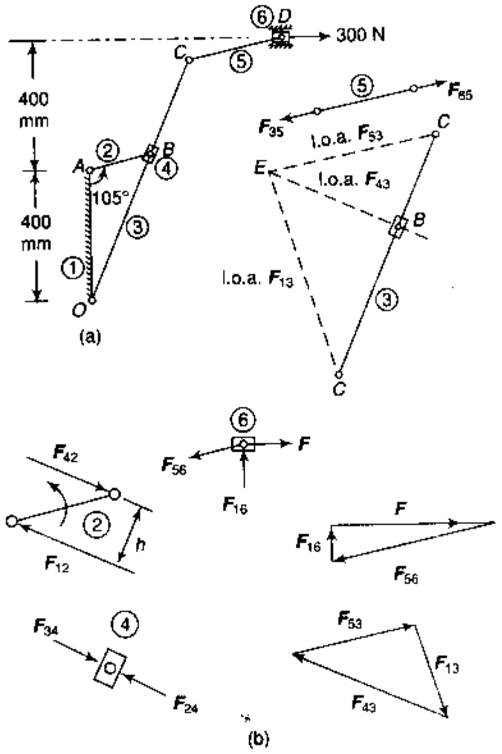


Fig. 12.15

Considering the equilibrium of the slider 4, the direction of $\mathbf{F}_{24}$ is known which is equal and opposite to $\mathbf{F}_{34}$. Considering the equilibrium of the link 2, the lines of action of $\mathbf{F}_{42}$ and $\mathbf{F}_{12}$ are drawn and the perpendicular distance between them is measured.

Then, torque on the link 2,

$T_2 = F_{42} \times h = 403 \times 120 = 48\ 360$ N counter-clockwise

Example 12.9 *A four-link mechanism is subjected to the following external forces (Fig.12.16 & Table 12.1): Determine the shaft torque $\mathbf{T}_2$ on the input link AB for static equilibrium of the mechanism. Also find the forces on the bearings A, B, C and D.*

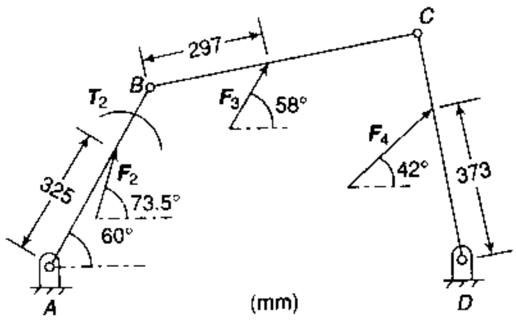


Fig. 12.16

Solution

The solution of the stated problem is worked out by

(i) graphical solution by using theorem of superposition, i.e., dividing the problem into subproblems by considering only one force on a member and ignoring the other forces on other members
(ii) a composite graphical solution
(iii) analytical solution

(i) Graphical Method by Superposition

Subproblem a (Fig. 12.17) Neglecting forces $\mathbf{F}_3$ and $\mathbf{F}_4$.

Table 12.1

Link	*Length*	*Force*	*Magnitude*	*Point of application force (r)*
AB (2)	500 mm	$\mathbf{F}_2$	80 ∠ 73.5°N	325 mm from *A*
AB (3)	660 mm	$\mathbf{F}_3$	144 ∠ 58°N	297 mm from B
AB (4)	560 mm	$\mathbf{F}_4$	60 ∠ 42°N	373 mm from *D*
AB (1)	1000 mm	-	(Fixed link)	

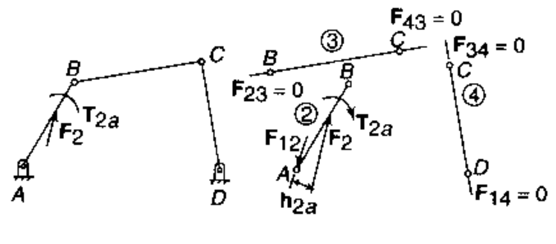


Fig. 12.17

Links 3 and 4 are both two-force members. Therefore, $\mathbf{F}_{43}$ can be along BC and $\mathbf{F}_{43}$, along DC. As $\mathbf{F}_{43}$ is to be equal and opposite of $\mathbf{F}_{34}$, both must be zero.

Also $F_{43} = F_{23} = F_{32} = 0$

Hence, the link 2 is in equilibrium under the action of two forces $\mathbf{F}_2$ and $\mathbf{F}_{12}$ ($F_{12} = F_2$) and torque $\mathbf{T}_{2a}$.

$T_{2a} = F_2 \times h_{2a} = 80 \times 0.325 \sin 13.5° = 6$ N.m clockwise

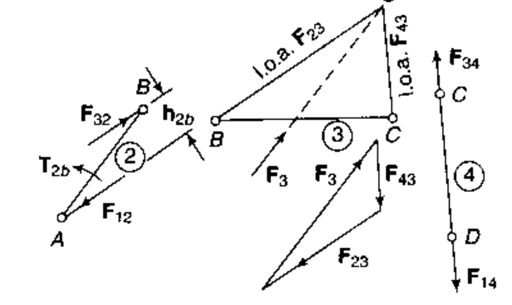


Fig. 12.18

Subproblem b (Fig. 12.18) Neglecting forces $\mathbf{F}_2$ and $\mathbf{F}_4$.

Link 4 is a two-force member.

$\therefore$ $\mathbf{F}_{34} = \mathbf{F}_{14}$, magnitudes unknown, directions parallel to DC

Link 3 is a three-force member in which $\mathbf{F}_3$ is completely known, only the direction of $\mathbf{F}_{43}$ is know (parallel to DC) and $\mathbf{F}_{23}$ is completely unknown. If the line of action of $\mathbf{F}_3$ and $\mathbf{F}_{43}$ meet at O, the line of action of $\mathbf{F}_{23}$ will be along OB. Draw the force triangle ($\mathbf{F}_3$ is completely known) by taking $\mathbf{F}_3$ to a suitable scale and two lines parallel to lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Mark arrowheads on $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ to know the directions.

$$F_{43} = 50 \text{ N}$$

Also, $F_{43} = F_{34} = \mathbf{F}_{14} = 50$ N

$$F_{23} = 113 \text{ N}$$

and $F_{23} = F_{32} = 113$ N

Link 2 is subjected to two forces and a torque $\mathbf{T}_{2b}$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_{2b} = F_{32} \times h_{2b} = 113 \times 0.16 = 18.1$ N.m counter-clockwise.

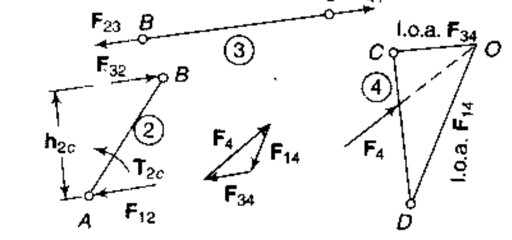


Fig. 12.19

Subproblem c (Fig. 12.19) Neglecting forces $\mathbf{F}_2$ and $\mathbf{F}_3$.

Link 4 is a three-force member in which only one force $\mathbf{F}_4$ is known. However, the line of action of $\mathbf{F}_{34}$ can be obtained from the equilibrium of the link 3 which is a two-force member. $\mathbf{F}_{34}$ will be equal and opposite to $\mathbf{F}_{43}$ which is along BC. If $\mathbf{F}_4$ and $\mathbf{F}_{34}$ intersect at O then the line of action of $\mathbf{F}_{14}$ will be along OD. Draw the force triangle ($\mathbf{F}_4$ is completely known) and obtain the magnitudes of forces $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

$$F_{14} = 34.8 \text{ N}$$

Also, $F_{34} = F_{43} = F_{23} = F_{32} = 34$ N

Link 2 is subjected to two forces and a torque $\mathbf{T}_{2c}$.

For equilibrium,

$\mathbf{F}_{12} = \mathbf{F}_{32}$

$T_{2c} = F_{32} \times h_{2c} = 34 \times 0.38 = 12.9$ N.m counter-clockwise.

Net crankshaft torque $= T_{2a} + T_{2b} + T_{2c}$
$= -6 + 18.1 + 12.9$
$= 25$ N.m counter-clockwise

To find the magnitudes of forces on the bearings, the results obtained in a, b and c have to be added vectorially as shown in Fig. 12.20.

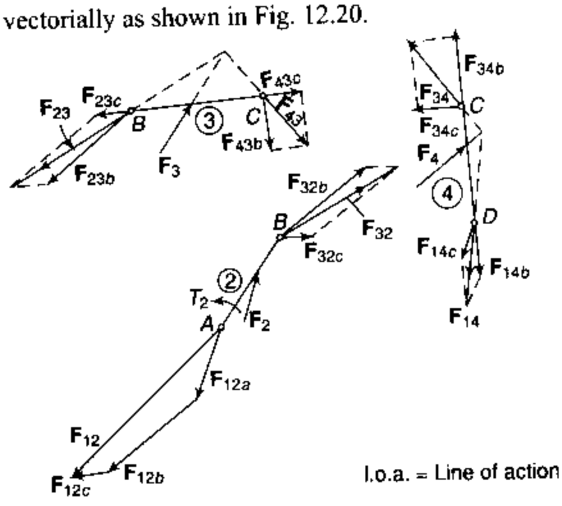


Fig. 12.20

$F_{14} = 80$ N
$F_{34} = F_{43} = 60$ N
$F_{23} = F_{32} = 137$ N
$F_{12} = 204$ N

(ii) Composite graphical solution

The problem can be solved by following the same procure as in examples 12.6 and 12.7. The solution is worked out in Fig. 12.21 which is self-explanatory. After obtaining the force $\mathbf{F}_{32}$, the resultant R' of this force with the force $\mathbf{F}_2$ can be obtained by drawing a force diagram. This resultant passes through the intersection of the lines of action of $\mathbf{F}_2$ and $\mathbf{F}_{23}$.

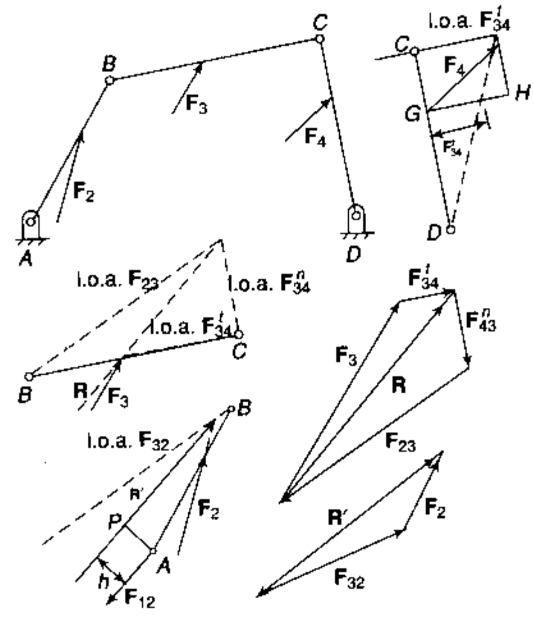


Fig. 12.21

$T = \mathbf{R}' \times h = \mathbf{F}_{12} \times h$
$= 208.8 \times 117 = 24\,430$ N.mm or 24.43 N.m

(iii) Analytical Method

First of all, determine the angular inclinations of the links BC and DC, i.e., angles β and ϕ. This may be done by drawing the configuration or by analytical means (section 4.2). Angles β and ϕ are found to be 10.3° and 100.4° (Fig. 12.22) respectively using analytical means.

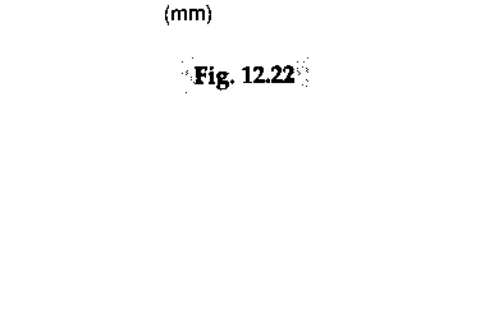


Fig. 12.22

Position vectors $\mathbf{AB} = 0.5\ \angle 60^\circ = 0.25\mathbf{i} + 0.433\mathbf{j}$

$\mathbf{BC} = 0.66\ \angle 10.3^\circ = 0.649\ \mathbf{i} + 0.118\ \mathbf{j}$

$\mathbf{DC} = 0.56\ \angle 100.4^\circ = -0.101\ \mathbf{i} + 0.551\ \mathbf{j}$

$\mathbf{AE} = 0.325\ \angle 60^\circ = 0.163\ \mathbf{i} + 0.281\ \mathbf{j}$

$\mathbf{BG} = 0.297\ \angle 10.3^\circ = 0.292\ \mathbf{i} + 0.053\ \mathbf{j}$

$\mathbf{DH} = 0.373 \angle 100.4^\circ = -0.0673\mathbf{i} + 0.367\mathbf{j}$

Force vectors $\mathbf{F}_2 = 80\ \angle 73.5^\circ = 22.72\ \mathbf{i} + 76.7\ \mathbf{j}$

$\mathbf{F}_3 = 144\ \angle 58^\circ = 76.31\ \mathbf{i} + 122.1\ \mathbf{j}$

$\mathbf{F}_4 = 60\ \angle 42^\circ = 44.59\ \mathbf{i} + 40.15\ \mathbf{j}$

Subproblem a

$$\mathbf{F}_{14} = -\mathbf{F}_2 = -80\ \angle 73.5^\circ$$
$$= 80\angle 253.5^\circ = -22.72\ \mathbf{i} - 76.7\ \mathbf{j}$$
$$T_{2a} = \mathbf{F}_2 \times \mathbf{AE} = (22.72\ \mathbf{i} + 76.7\ \mathbf{j}) \times (0.163\ \mathbf{i} + 0.281\ \mathbf{j})$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 22.72 & 76.7 & 0 \\ 0.163 & 0.281 & 0 \end{vmatrix}$$
$$= 22.72 \times 0.281 - 76.7 \times 0.163$$
$$= -6.12\ \text{N.m}$$

Subproblem b As the link BC is in static equilibrium, the resultant forces and moments acting on it are zero.

Taking moments of the forces about point B,

$$M_b = \mathbf{F}_3 \times \mathbf{BG} + \mathbf{F}_{43} \times \mathbf{BC} = \mathbf{0} \qquad \text{(i)}$$

As the direction of $\mathbf{F}_{43}$ is along DC if force $\mathbf{F}_4$ is ignored,

$\therefore\ \mathbf{F}_{43} = F_{43}\ \angle 100.4^\circ = -0.181\ F_{43}\ \mathbf{i} + 0.983\ F_{43}\mathbf{j}$

Inserting the values of vectors in (i),

$(76.31\ \mathbf{i} + 122.1\mathbf{j}) \times (0.292\ \mathbf{i} + 0.053\mathbf{j})$
$+(-0.181 F_{43}\mathbf{i} + 0.983 F_{43}\mathbf{j}) \times (0.649\mathbf{i} + 0.118\mathbf{j}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 76.3 & 122.1 & 0 \\ 0.292 & 0.053 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.181F_{43} & 0.983F_{43} & 0 \\ 0.649 & 0.118 & 0 \end{vmatrix} = 0$$

$-31.61 - 0.659\ F_{43} = 0$

$F_{43} = -48$

Thus

$\mathbf{F}_{43} = -48\angle 100.4^\circ = 48\angle 280.4^\circ = 8.66\ \mathbf{i} - 47.1\mathbf{j}$

$\mathbf{F}_{14} = -\mathbf{F}_{34} = \mathbf{F}_{43} = 48\angle 280.4^\circ = 8.66\ \mathbf{i} - 47.1\mathbf{j}$

Similarly, the net force on the link 3,

$\mathbf{F}_{23} + \mathbf{F}_3 + \mathbf{F}_{43} = 0$

or $\mathbf{F}_{23} + (76.31\ \mathbf{i} + 122.1\mathbf{j}) + (8.66\ \mathbf{i} - 47.1\mathbf{j}) = 0$

or $\mathbf{F}_{23} + 84.97\ \mathbf{i} + 75\mathbf{j} = 0$

or $\mathbf{F}_{23} = -84.97\ \mathbf{i} - 75\mathbf{j}$ or $113.3\ \angle 221.4^\circ$

or $\mathbf{F}_{32} = 113.3\ \angle 41.4^\circ = 84.97\ \mathbf{i} + 75\mathbf{j}$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = -84.97\ \mathbf{i} - 75\ \mathbf{j}$

$T_{2b} = \mathbf{F}_{32} \times AB\angle 60^\circ = (84.97\ \mathbf{i} + 75\ \mathbf{j}) \times (0.25\ \mathbf{i} + 0.433\ \mathbf{j})$

$= 18\ \text{N.m}$

Subproblem c As the link DC is in static equilibrium, no forces and no moments are acting on it. Taking moments of the forces about point D,

$$M_d = \mathbf{F}_4 \times \mathbf{DH} + \mathbf{F}_{34} \times \mathbf{DC} = \mathbf{0} \qquad \text{(ii)}$$

As the direction of $\mathbf{F}_{34}$ is along BC if the force $\mathbf{F}_3$ is ignored,

$\therefore\ \mathbf{F}_{34} = F_{34}\ \angle 10.3^\circ = 0.984\ F_{34}\ \mathbf{i} + 0.179\ F_{34}\mathbf{j}$

Inserting the values of vectors in (ii),

$(44.59\ \mathbf{i} + 40.15\ \mathbf{j}) \times (-0.0673\ \mathbf{i} + 0.367\ \mathbf{j})$
$+(0.984 F_{34}\mathbf{i} + 0.179 F_{34}\mathbf{j}) \times (-0.101\mathbf{i} + 0.551\mathbf{j}) = 0$

or

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 44.59 & 40.15 & 0 \\ -0.0673 & 0.367 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.984F_{43} & 0.179F_{43} & 0 \\ -0.101 & 0.551 & 0 \end{vmatrix} = 0$$

or $19.067 + 0.56\ F_{34} = 0$

$F_{34} = -34$

Thus

$\mathbf{F}_{34} = -34\angle 10.3^\circ = 34\angle 190.3^\circ = -33.45\mathbf{i} - 6.08\mathbf{j}$

Net force on the link 4,

$\mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_{14} = 0$

or $(-33.45\ \mathbf{i} - 6.08\ \mathbf{j}) + (44.59\ \mathbf{i} + 40.15\ \mathbf{j}) + \mathbf{F}_{14} = 0$

or $11.14\ \mathbf{i} + 34.07\ \mathbf{j} + \mathbf{F}_{14} = 0$

or $\mathbf{F}_{14} = -11.14\ \mathbf{i} - 34.07\mathbf{j}$ or $35.8\ \angle 251.9^\circ$

Now. $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 33.45\ \mathbf{i} + 6.08\mathbf{j}$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = -33.45\ \mathbf{i} - 6.08\ \mathbf{j}$

$T_{2c} = \mathbf{F}_{32} \times AB\angle 60^\circ = (33.45\ \mathbf{i} + 6.08\ \mathbf{j}) \times (0.25\ \mathbf{i} + 0.433\ \mathbf{j})$

$= 12.96\ \text{N.m}$

Net crankshaft torque $= T_{2a} + T_{2b} + T_{2c}$

$= -6.12 + 18 + 12.96$

$=$ <u>24.84 N.m</u> counter-clockwise

Forces on the bearings

On D, $\mathbf{F}_{14} = (8.66\mathbf{i} - 47.1\ \mathbf{j}) + (-11.14\mathbf{i} - 34.07\mathbf{j})$

$= -2.48\ \mathbf{i} - 81.17\ \mathbf{j}$

$= 81.2\ \angle 268.2^\circ\text{N}$

or it can be stated as $\mathbf{F}_{41} = 81.2\ \angle 88.2^\circ\text{N}$

On C, $\mathbf{F}_{43} = (8.66\ \mathbf{i} - 47.1\ \mathbf{j}) + (33.45\ \mathbf{i} + 6.08\ \mathbf{j})$

$= 41.45\ \mathbf{i} - 41.02\ \mathbf{j}$

$= 58.8\ \angle 315.8^\circ\text{N}$

On B, $\mathbf{F}_{23} = (-84.97\,\mathbf{i} - 75\,\mathbf{j}) + (-33.45\,\mathbf{i} - 6.08\,\mathbf{j})$
$= 118.42\,\mathbf{i} - 81.08\,\mathbf{j}$
$= 143.5\ \angle 214.4°\text{N}$

On A, $\mathbf{F}_{12} = (-22.72\,\mathbf{i} - 76.7\,\mathbf{j}) + (-84.97\,\mathbf{i} - 75\,\mathbf{j})$
$+ (-33.45\,\mathbf{i} - 6.08\,\mathbf{j})$
$= -141.14\,\mathbf{i} - 157.78\,\mathbf{j}$
$= 211.7\angle 228.2°\text{N}$

Example 12.10 ***In a four-link mechanism shown in Fig. 12.23(a), torque T_3 and T_4 have magnitudes of 30N.mand20N.mrespectively. The link lengths are AD = 800 mm, AB = 300 mm, BC = 700 mm and CD = 400 mm. For the static equilibrium of the mechanism, determine the required input torque T_2.***

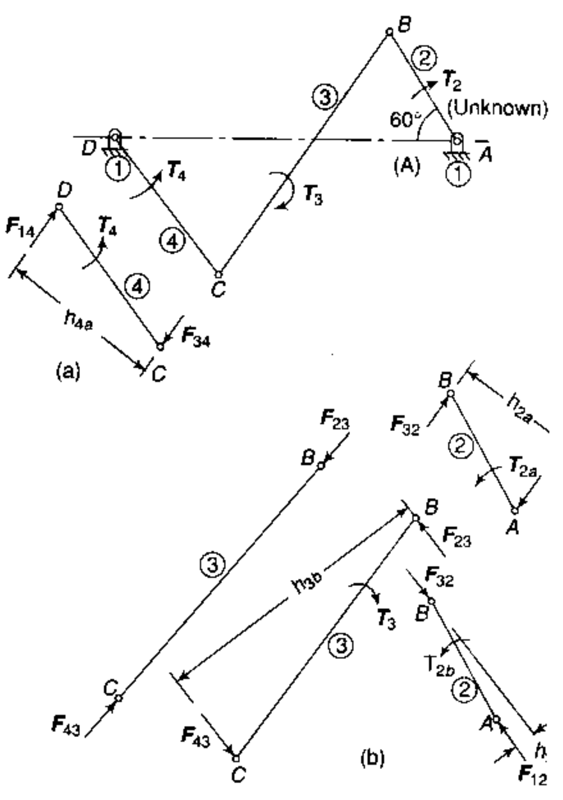


Fig. 12.23

Solution The solution of the stated problem can be obtained by superposition of the solutions of subproblems *a* and *b*.

Subproblem a [Fig. 12.23(a)] Neglecting torque $\mathbf{T}_3$

Torque $\mathbf{T}_4$ on the link 4 is balanced by a couple having two equal, parallel and opposite forces at C and D. As the link 3 is a two-force member, $\mathbf{F}_{43}$ and therefore, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ will be parallel to BC.

$$F_{34} = F_{14} = \frac{T_4}{h_{4a}} = \frac{20}{0.383} = 52.2 \text{ N}$$

and $F_{34} = F_{43} = F_{23} = F_{32} = F_{12} = 52.2$ N

$T_{2a} = F_{32} \times h_{2a} = 52.2 \times 0.274 = 14.3$ N.m counter-clockwise.

Subproblem b [Fig. 12.23(b)] Neglecting torque $\mathbf{T}_4$.

$\mathbf{F}_{43}$ is along CD. The diagram is self-explanatory.

$$F_{43} = F_{23} = \frac{T_3}{h_{3b}} = \frac{30}{0.67} = 44.8 \text{ N}$$

$F_{23} = F_{32} = F_{12} = 44.8$ N

$T_{2b} = F_{32} \times h_{2b} = 44.8 \times 0.042 = 1.88$ N.m

counter-clockwise.

$T_2 = T_{2a} + T_{2b} = 14.3 + 1.88 = \underline{16.18 \text{ N}}$

counter-clockwise

Example 12.11 ***Figure 12.24 shows a schematic diagram of an eight-link mechanism. The link lengths are***

AB = 450 mm	*OF = FC = 250 mm*
AC = 300 mm	*CG = 150 mm*
BD = 400 mm	*HG = 600 mm*
BE = 200 mm	*QH = 300 mm*

Determine the required shaft torque on the link 8 for static equilibrium against an applied load of 400 N on the link 3.

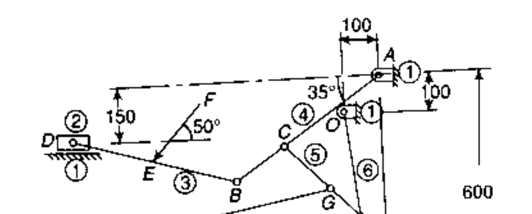


Fig. 12.24

Solution Links 2, 6 and 7 are two-force members. Since their lines of action can easily be visualised, it is not necessary to draw their free-body diagrams. Links 3, 4 and 5 are three-force members and 8 is a member with two forces and a torque.

Slider 2 is a two-force member. If friction is neglected, the forces on it $\mathbf{F}_{12}$ and $\mathbf{F}_{32}$ must act perpendicular to the guide path.

Considering the link 3, concurrency point can be found from the lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}$, and thus the line of action of $\mathbf{F}_{43}$ is established.

The equilibrium of the link 4 cannot be considered at this stage as the line of action of only one force $\mathbf{F}_{34}$ is known (from $\mathbf{F}_{43}$).

Taking the link 5 which is a three-force member, the line of action of force at F is along OF and of force at G along HG. Establishing the point of concurrency from these two forces, the line of action of force at C, i.e., of the force $\mathbf{F}_{45}$ is known.

Now, take the link 4 and determine the line of action of the force at A since the lines of action of forces at B and C are known.

Force $\mathbf{F}_{78}$ is along HG and an equal, parallel and opposite force $\mathbf{F}_{18}$ also acts on the link 8.

Now, the lines of action of all the forces are known. To determine the torque on the link 8, proceed as follows:

Construct a force diagram for the forces on the link 3 ($\mathbf{F}$ is completely known) and find $\mathbf{F}_{43}$ (thus $\mathbf{F}_{34}$ is known).

Draw a force diagram for the forces on the member 4 ($\mathbf{F}_{34}$ is complete known) and find $\mathbf{F}_{54}$ (thus $\mathbf{F}_{45}$ is known).

Draw a force diagram for the forces on the member 5 ($\mathbf{F}_{45}$ is complete known) and find $\mathbf{F}_{75}$ (thus $\mathbf{F}_{57}$ is known).

Now $F_{57} = F_{87} = F_{78} = F_{18}$

$F_8 = F_{78} \times h = 75 \times 240 = \underline{18\ 000 \text{ N.mm}}$

or $\underline{18 \text{ N.m}}$ clockwise

12.9 PRINCIPLE OF VIRTUAL WORK

The principle of virtual (imaginary) work can be stated as 'the work done during a virtual displacement from the equilibrium is equal to zero'. Virtual displacement may be defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it into free bodies.

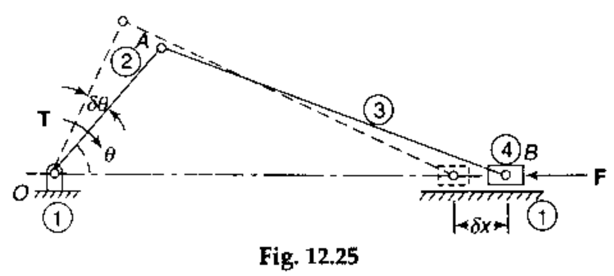


Fig. 12.25

Consider a slider-crank mechanism shown in Fig. 12.25. It is acted upon by the external piston force **F**, the external crankshaft torque **T** and the force at the bearings. As the crank rotates through a small angular displacement $\delta\theta$, the corresponding displacement of the piston is δx. the various forces acting on the system are

- Bearing reaction at O (performs no work)
- Force of cylinder on piston, perpendicular to piston displacement (produces no work)
- Bearing forces at A and B, being equal and opposite (AB is a two-force member), no work is done
- Work done by torque $T = T\delta\theta$
- Work done by force $F = F\,\delta x$

Work done is positive if a force acts in the direction of the displacement and negative if it acts in the opposite direction.

According to the principle of virtual work,

$$W = T\,\delta\theta + F\,\delta x = 0 \tag{12.7}$$

As virtual displacement must take place during the same interval δt,

$$\therefore \qquad T\frac{d\theta}{dt} + F\frac{dx}{dt} = 0$$

or

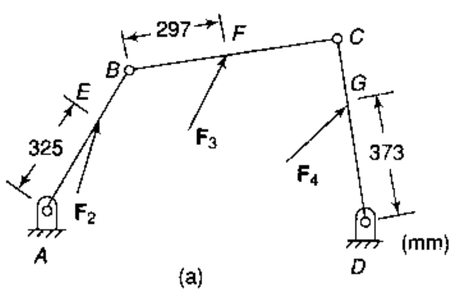
$$T\omega + Fv = 0 \tag{12.8}$$

where ω is the angular velocity of the crank and v, the linear velocity of the piston.

$$T = -\frac{F}{\omega}v$$

The negative sign indicates that for equilibrium, **T** must be applied in the opposite direction to the angular displacement.

Example 12.12 *Solve Example 12.9 by using the principle of virtual work.*



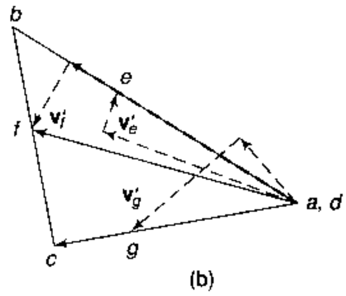


Fig. 12.26

Solution Assume that the line AB has an instantaneous angular velocity of ω rad/s counter-clockwise. Then $v_b = 0.5\,\omega$ m/s.

From the configuration diagram [Fig. 12.26(a)], draw the velocity diagram [Fig. 12.26(b)]. Locate the points E, F and G on the velocity diagram and locate the velocity vectors for the same. Take their components parallel and perpendicular to the direction of forces.

$v'_e = 0.0745$ rad/s (parallel to $\mathbf{F}_2$)

$v'_f = 0.124$ rad/s (parallel to $\mathbf{F}_3$)

$v'_g = 0.205$ rad/s (parallel to $\mathbf{F}_4$)

Assuming T to be counter-clockwise and applying the principle of virtual work,

$T \times \omega + F_2 \times 0.0745\omega - F_3 \times 0.124\omega - F_4 \times 0.205\omega = 0$

or $\quad T + 80 \times 0.0745 - 144 \times 0.124 - 60 \times 0.205 = 0$

or $\quad T = -6 + 17.3 + 12.3$

$= 23.5$ N.m counter-clockwise

FRICTION IN MECHANISMS

When two members of a mechanism move relative to each other, friction occurs at the joints. The presence of friction increases the energy requirements of a machine.

Friction at the bearing is taken into account by drawing friction circles and at the sliding pairs by considering the angle of friction (Refer sections 8.13 and 8.14).

Example 12.13 *In a four-link mechanism ABCD,*

AB = 350 mm, AD = 700 mm

BC = 500 mm, DE = 150 mm

CD = 400 mm, ∠DAB = 60° (AD is the fixed link)

A force of 35 N (F_4) acts at E on link DC as shown in Fig. 12.27a. Determine the force on the link AB required at the midpoint in the direction shown in the diagram for the static equilibrium of the mechanism. The coefficient of friction is 0.4 for each revolving pair. Assume impending motion of AB to be counter-clockwise. The radius of each journal is 50 mm.

Also, find the torque on AB for its impending clockwise motion. (A very high value of coefficient of friction has been assumed to obtain a clear diagram).

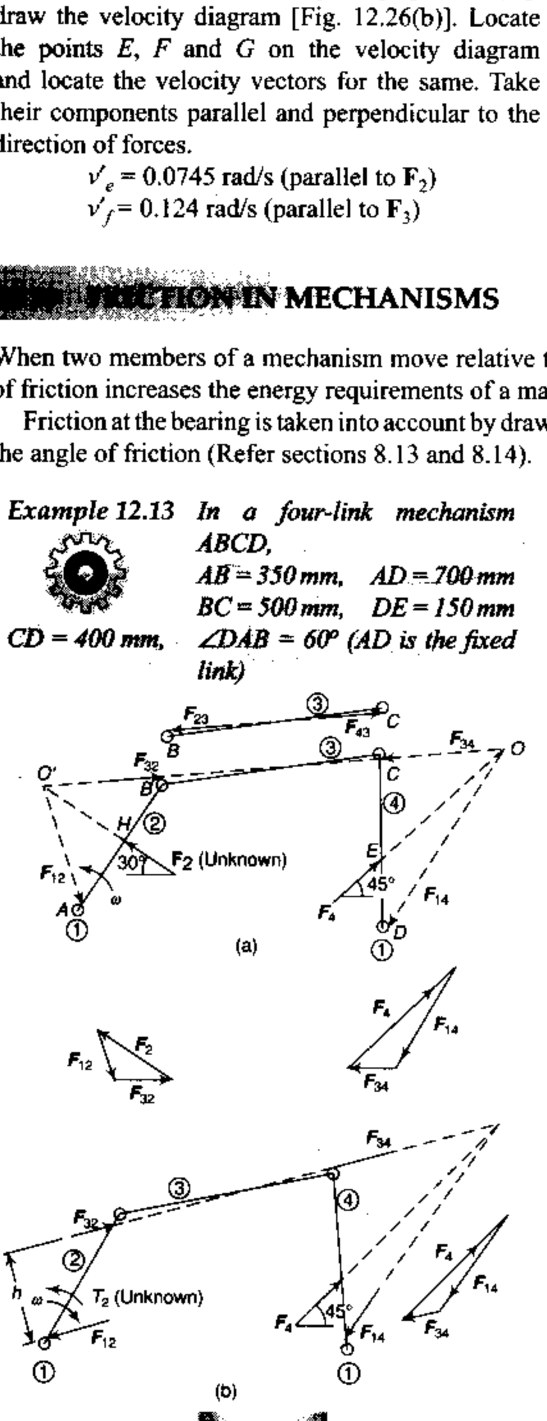


Fig. 12.27

Solution Radius of friction circle at each joint = μr = $0.4 \times 50 = 20$ mm.

For the counter-clockwise rotation of link AB, DC also rotates counter-clockwise; $\angle ABC$ is deceasing and $\angle BCD$ increasing.

Initially, neglect the friction at the journal bearings and find the directions of different forces by finding points of concurrency and drawing force triangles (not shown in the diagram).

Considering the link 3, at its end C, $\angle BCD$ is increasing and thus it rotates clockwise relative to the link 4. Therefore, $\mathbf{F}_{43}$ must form a counter-clockwise friction couple. At the end B, $\angle ABC$ is decreasing and thus rotates clockwise relative to the link 2. Therefore, $\mathbf{F}_{23}$ forms a counter-clockwise friction couple. The friction axis for the coupler BC is the common tangent to the two friction circles.

Now, consider the link 4. The line of action of the force $\mathbf{F}_{34}$ will be opposite to that of $\mathbf{F}_{43}$. Intersection of this line with the line of action of $\mathbf{F}_4$ gives the point of concurrency O for the forces acting on the link 4. As the link 4 rotates counter-clockwise, the tangent to the friction circle at D drawn from point O is such that a clockwise friction couple is obtained.

By drawing a force triangle for the forces acting on link 4 ($\mathbf{F}_4$ is completely known), $\mathbf{F}_{34}$ is obtained.

$$F_{34} = F_{43} = F_{23} = F_{32}$$

The point of concurrency for the forces acting on the link 2 is at O' which is the intersection of $\mathbf{F}_{32}$ and $\mathbf{F}_2$. As the link 1 rotates counter-clockwise, draw a tangent to the friction circle at A from O' such that a clockwise friction couple is obtained.

Draw a force diagram for the forces acting on the link 2 ($\mathbf{F}_{32}$ is completely known) and obtain the value of $\mathbf{F}_2$.

$$F_2 = 20.3 \text{ N}$$

When the motion of AB is clockwise, DC also moves clockwise. For the equilibrium of the link 4, the friction couples at D and C are to be counter-clockwise. For the equilibrium of the link 2, friction couples at A and B are also to be counter-clockwise. Obtain $\mathbf{F}_{32}$ in the manner discussed above and shown in Fig. 12.27(b) $\mathbf{F}_{12}$ will be equal, parallel and opposite to $\mathbf{F}_{32}$.

$$T_2 = F_{32} \times h = 8.6 \times 208 = 1789 \text{ N.mm}$$

or <u>1.789 N. m</u>

Example 12.14 *Find the minimum value of force F_5 to be applied for the static equilibrium of the follower of Example 12.2 if the friction is also considered of the sliding bearings at B and C. Assume the coefficient of friction as 0.15. Ignore the thickness of the follower.*

Solution When a force analysis with friction is to be made, it is always convenient to seek a rough solution of the problem first without friction. This may be obtained by drawing freehand sketches. The purpose is to know the direction-sense of the normal reactions at B and C as these have to be combined with the friction forces at the sliders. Adopting the procedure of Example 12.2, the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ at the bearings are found to be towards right.

As the force $\mathbf{F}_5$ required for the static equilibrium is to be the least, i.e., any force smaller than that will make the follower move down due to the applied force. Thus, the impending motion of the follower is downwards. (If it is desired to have the maximum force for the static equilibrium, any force greater than that will make the follower move up and the impending motion of the follower will be upwards).

Now, as the impending motion of the follower is downwards, the friction forces at the bearings are upwards. Combining these forces with the reaction forces which are towards right, the lines of action of both the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ are tilted through an angle ϕ given by

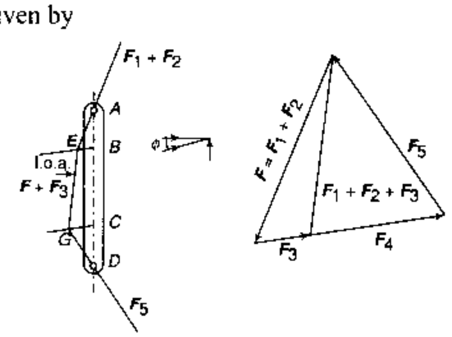


Fig. 12.28

$$\mu = 0.15$$

or $\tan\phi = 0.15$

or $\phi = 8.5°$

On knowing the new lines of action of $\mathbf{F}_3$ and $\mathbf{F}_4$ [Fig. 12.28(a)], the exact solution can be easily obtained as before [Fig. 12.28(b)]. The values obtained are

Magnitude of $\mathbf{F}_3 = 14.5$ N
Magnitude of $\mathbf{F}_4 = 35.5$ N
Magnitude of $\mathbf{F}_5 = 51$ N

Example 12.15 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.29a, find the maximum input torque T_2 required for a force of 300 N on the slider D. Angle θ is 105°. Coefficient of friction $\mu = 0.15$ for each sliding pair.*

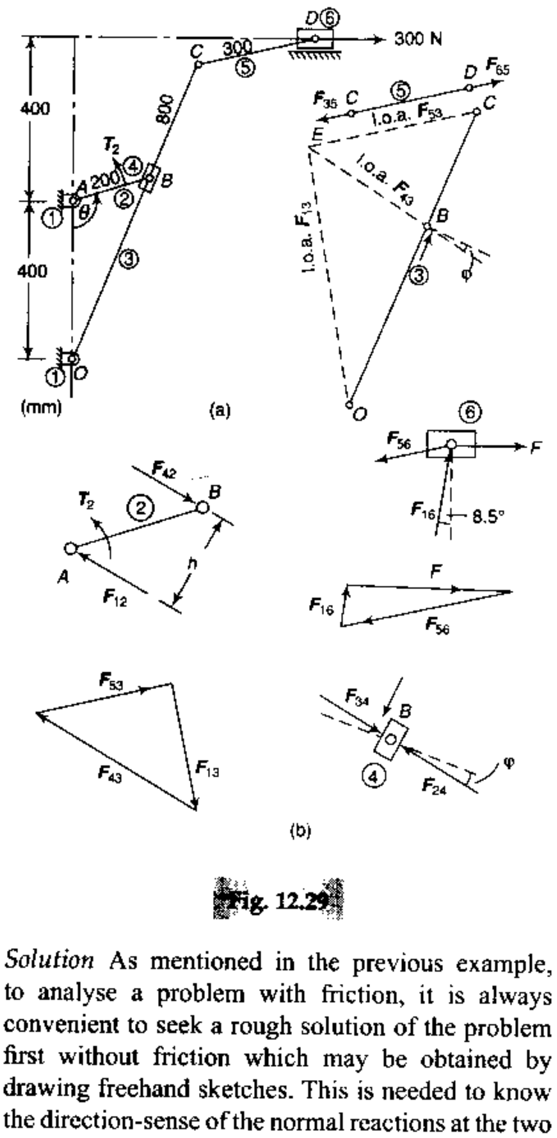


Fig. 12.29

Solution As mentioned in the previous example, to analyse a problem with friction, it is always convenient to seek a rough solution of the problem first without friction which may be obtained by drawing freehand sketches. This is needed to know the direction-sense of the normal reactions at the two sliders which are to be combined with the friction forces.

As the torque required for the static equilibrium is to be the maximum, i.e., any torque more than that will make the slider at D move left. Thus, the impending motion of the slider D is to the left.

Now,

$$\mu = 0.15 \quad \text{or} \quad \tan\phi = 0.15$$

or $\phi = 8.5°$

Solving the problem first without friction,

Slider at D or the link 6 is a three-force member. Lines of action of the forces are

- **F**, as given
- $\mathbf{F}_{56}$ along CD, as link 5 is a two-force member
- $\mathbf{F}_{16}$, normal reaction, perpendicular to slider motion

Draw the force diagram and determine the direction sense of forces $\mathbf{F}_{56}$ and $\mathbf{F}_{16}$ from it (the diagrams may not be to scale). From the force $\mathbf{F}_{56}$, the directions of forces $\mathbf{F}_{65}$, $\mathbf{F}_{35}$ and $\mathbf{F}_{53}$ are known. Now link 3 is a three-force member. Lines of action of the forces are

- $\mathbf{F}_{53}$, known completely through C
- $\mathbf{F}_{43}$, perpendicular to slider motion through B
- $\mathbf{F}_{13}$, unknown through A.

As the lines of action of forces acting through B and C are known, the line of action of $\mathbf{F}_{13}$ through A must also pass through the point of intersection of the other two forces. Find the sense of the direction of force $\mathbf{F}_{43}$ by drawing the force triangle.

After obtaining the sense of direction of the normal forces $\mathbf{F}_{16}$ (upwards) and $\mathbf{F}_{43}$ (towards left), solve the problem by considering the force of friction also. Now the diagrams must be to the scale.

The force of friction at the slider D is towards right as the impending motion of the slider is towards the left. Combining this force with the normal force $\mathbf{F}_{16}$, it is tilted towards left as shown in the figure. Now draw the force triangle by modifying the line of action of force $\mathbf{F}_{16}$. Repeat the above procedure and obtain magnitude as well the direction of the force $\mathbf{F}_{53}$.

The motion of the slider 4 on the link 3 is upwards for impending motion of the slider D towards left. It implies that the motion of the link 3 relative to the link 4 is downwards. Thus, force of friction on the link 3 is upwards (on slider it is downwards). Combining this with the normal force $\mathbf{F}_{43}$ which is towards left, the force $\mathbf{F}_{43}$ is tilted through an angle ϕ as shown in the figure. Now again draw the force triangle with the modified direction of the force $\mathbf{F}_{43}$ for the forces on the link 3 and obtain the magnitude of this force also.

Now,

$$F_{34} = F_{43}$$

As the slider B is a two-force member with forces $\mathbf{F}_{24}$ and $\mathbf{F}_{34}$. Therefore,

$$F_{34} = F_{24} = F_{42} = F_{12}$$

Thus, as the link 2 is acted upon by two forces and a torque,

$$T = F_{42} \times h = 437 \times 147 = 64240 \text{ N.m}$$

$$= \underline{64.24 \text{ N.m}} \text{ counter-clockwise}$$

Example 12.16 *Solve Example 8.28 using graphical method. Take coefficient of friction for the journals as 0.4 instead of 0.05.* ***(A fictitious high value of coefficient of friction is taken so that friction circles of reasonable diameter may be drawn on a smaller scale).***

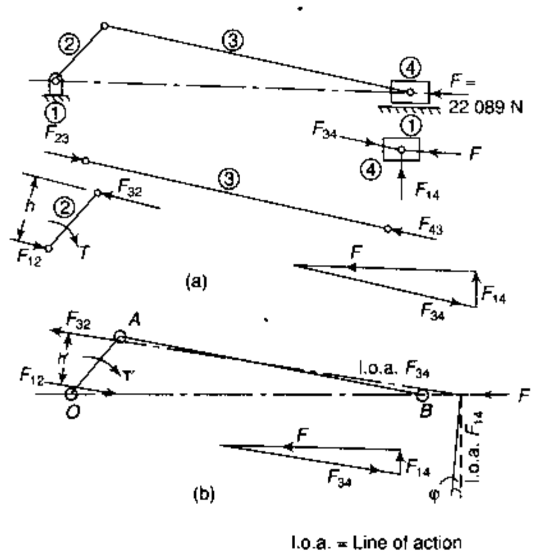


Fig. 12.30

Solution Figure 12.30(a) shows the solution of the problem neglecting the friction. From the force triangle for the forces on the slider,

$$F_{34} = 22\ 500 \text{ N}$$

Now,

$$F_{34} = -F_{43} = F_{23} = F_{32}$$

$$T = F_{23} \times h = 22\ 500 \times 0.261$$

$$= 5872.5 \text{ N.m clockwise}$$

When friction is considered [Fig.12.30(b)],

$$\text{Radius of friction circle at } O = 0.4 \times \frac{140}{2} = 28 \text{ mm}$$

$$\text{Radius of friction circle at } A = 0.4 \times \frac{120}{2} = 24 \text{ mm}$$

$$\text{Radius of friction circle at } B = 0.4 \times \frac{80}{2} = 16 \text{ mm}$$

As the crank moves counter-clockwise, $\angle OAB$ decreases. AB rotates clockwise relative to OA. Thus, tangent at A is to be such that a counter-clockwise friction couple is obtained.

At B, $\angle OBA$ is increasing. Therefore, BA rotates clockwise relative to the piston. Thus, the tangent to the friction circle is to be such that it gives a counter-clockwise friction couple.

For the sliding pair, $\varphi = \tan^{-} 00.7 = 4°$

The point of intersection of $\mathbf{F}_{34}$ and $\mathbf{F}$ gives the point of concurrency for the forces on the slider. Force $\mathbf{F}_{14}$, i.e., the reaction of the guide, is inclined to the perpendicular to the slider path, and passes through the point of concurrency.

By drawing a force triangle for the forces acting on the slider, $\mathbf{F}_{34}$ is obtained.

The force at A is equal, parallel and opposite to $\mathbf{F}_{32}$ and tangent to the friction circle such that a clockwise friction couple is obtained.

$$T' = \mathbf{F}_{32} \times h' = 22\,200 \times 0.202 = 4484 \text{ N.m clockwise}$$

Summary

1. A pair of action and reaction forces which constrain two connected bodies to behave in a particular manner are known as *constraint forces* whereas forces acting from outside on a system of bodies are called *applied forces*.
2. A member under the action of two forces will be in equilibrium if the forces are of the same magnitude, act along the same line and are in opposite directions.
3. A member under the action of three forces will be in equilibrium if the resultant of the forces is zero and the lines of action of the forces intersect at a point, known as the *point of concurrency*.

4. A member under the action of two forces and an applied torque is in equilibrium if the forces are equal in magnitude, parallel in direction and opposite in sense and the forces form a couple which is equal and opposite to the applied torque.
5. The force exerted by the member *i* on the member *j* is represented by F_{ij}.
6. A *free-body diagram* is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.
7. In linear systems, if a number of loads act on a system of forces, the net effect is equal to the superposition of the effects of the individual loads taken one at a time. A linear system is one in which the output force is directly proportional to the input force, i.e., in mechanisms in which coulomb or dry friction is neglected.
8. The principle of virtual (imaginary) work can be stated as 'the work done during a virtual displacement from the equilibrium is equal to zero'. Virtual displacement may be defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it into free bodies.
9. Friction at the bearing is taken into account by drawing friction circles and at the sliding pairs by considering the angle of friction.

Exercises

1. What do you mean by applied and constraint forces? Explain.
2. What are conditions for a body to be in equilibrium under the action of two forces, three forces and two forces and a torque?
3. What are free-body diagrams of a mechanism? How are they helpful in finding the various forces acting on the various members of the mechanism?
4. Define and explain the superposition theorem as applicable to a system of forces acting on a mechanism.
5. What is the principle of virtual work? Explain.
6. How is the friction at the bearings and at sliding pairs of a mechanism is taken into account?
7. The dimensions of a four-link mechanism are: AB = 400 mm, BC = 600 mm, CD = 500 mm, AD = 900 mm, and $\angle DAB = 60°$. AD is the fixed link. E is a point on the link BC such that BE = 400 mm and CE = 300 mm (BEC clockwise)
   A force of 150 $\angle 45°$ N acts on DC at a distance of 250 mm from D. Another force of magnitude 100 $\angle 180°$ N acts at point E. Find the required input torque on the link AB for static equilibrium of the mechanism. (4.6 N.m clockwise)
8. Determine the required input torque on the crank of a slider-crank mechanism for the static equilibrium when the applied piston load is 1500 N. The lengths of the crank and the connecting rod are 40 mm and 100 mm respectively and the crank has turned through 45° from the inner-dead centre. (55 N.m)
9. Find the torque required to be applied to link AB of the linkage shown in Fig. 12.31 to maintain the static equilibrium. (8.85 N.m)

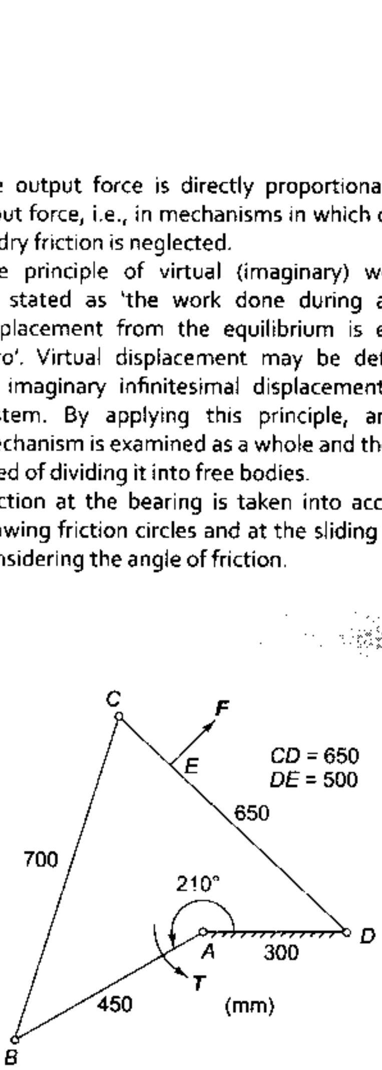


Fig. 11.31

10. Determine the torque required to be applied to the link OA for the static equilibrium of the mechanism shown in Fig. 12.32. (30.42 N.m)

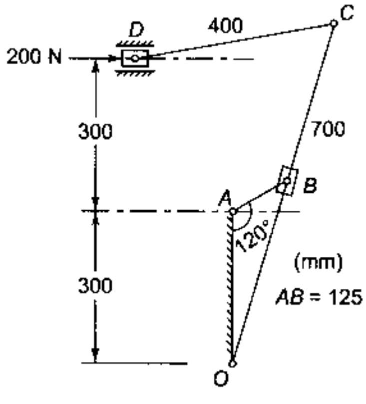


Fig. 12.32

11. For the mechanism shown in Fig. 12.33, find the required input torque for the static equilibrium. The lengths *OA* and *AB* are 250 mm and 650 mm respectively. *F* = 500 N. (68 N.m clockwise)

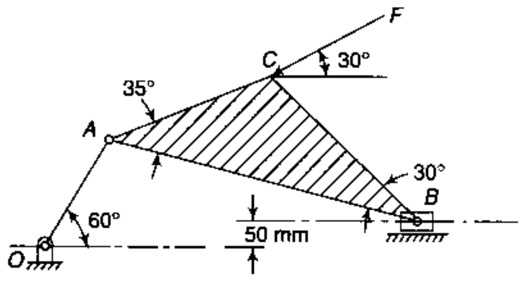


Fig. 12.33

12. For the static equilibrium of the mechanism of Fig. 12.34, find the required input torque. The dimensions are
AB = 150 mm, *BC* = *AD* = 500 mm, *DC* = 300 mm, *CE* = 100 mm and *EF* = 450 mm.
(45.5 N.m clockwise)

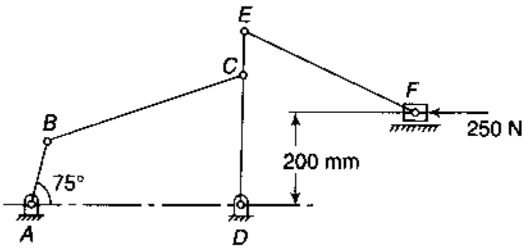


Fig. 12.34

13. Determine the torque to be applied to the link *AB* of a four link mechanism shown in Fig. 12.35 to maintain static equilibrium at the given position.
(44 N.m)

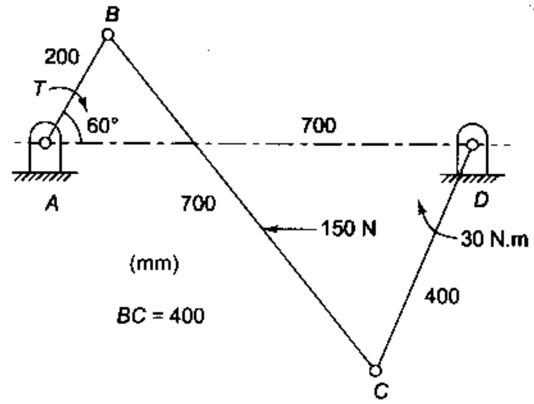


Fig. 12.35

14. A two-cylinder engine shown in Fig. 12.36 is in static equilibrium. The dimensions are *OA* = *OB* = 50 mm, *AC* = *BD* = 250 mm, ∠*AOB* = 90°. Determine the torque on the crank *OAB*.
(106 N.m clockwise)

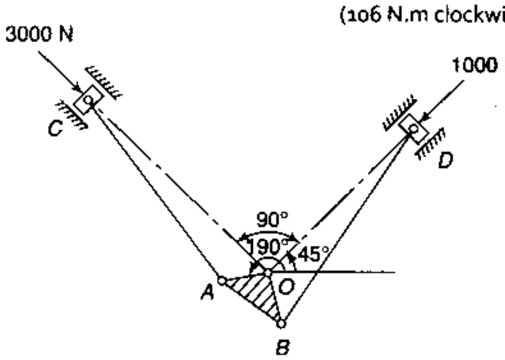


Fig. 12.36

13 DYNAMIC FORCE ANALYSIS

Introduction

Dynamic forces are associated with accelerating masses. As all machines have some accelerating parts, dynamic forces are always present when the machines operate. In situations where dynamic forces are dominant or comparable with magnitudes of external forces and operating speeds are high, dynamic analysis has to be carried out. For example, in case of rotors which rotate at speeds more than 80 000 rpm, even the slightest eccentricity of the centre of mass from the axis of rotation produces very high dynamic forces. This may lead to vibrations, wear, noise or even machine failure.

13.1 D'ALEMBERT'S PRINCIPLE

D'Alembert's principle states that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.

Inertia is a property of matter by virtue of which a body resists any change in velocity.

$$\text{Inertia force } \mathbf{F}_i = -m\,\mathbf{f}_g \tag{13.1}$$

where m = mass of body

$\mathbf{f}_g$ = acceleration of centre of mass of the body

The negative sign indicates that the force acts in the opposite direction to that of the acceleration. The force acts through the centre of mass of the body.

Similarly, an inertia couple resists any change in the angular velocity.

Inertia couple,

$$\mathbf{C}_i = -I_g \alpha \tag{13.2}$$

where I_g = moment of inertia about an axis passing through the centre of mass G and perpendicular to plane of rotation of the body

α = angular acceleration of the body

Let $\Sigma\mathbf{F} = \mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, etc. = external forces on the body

and $\Sigma\mathbf{T} = \mathbf{T}_{g1}, \mathbf{T}_{g2}, \mathbf{T}_{g3}$, etc. = external torques on the body about the centre of mass G.

According to D' Alembert's principle, the vector sum of forces and torques (or couples) has to be zero, i.e.,

$$\Sigma\mathbf{F} + \mathbf{F}_i = 0 \tag{13.3}$$

and

$$\Sigma \mathbf{T} + \mathbf{C}_i = 0 \tag{13.4}$$

These equations are similar to the equation of a body in static equilibrium, i.e., $\Sigma\mathbf{F} = 0$ and $\Sigma\mathbf{T} = 0$.

This suggests that first the magnitudes and the directions of inertia forces and couples can be determined, after which they can be treated just like static loads on the mechanism. Thus, a dynamic analysis problem is reduced to one requiring static analysis.

13.2 EQUIVALENT OFFSET INERTIA FORCE

In plane motions involving accelerations, the inertia force acts on a body through its centre of mass. However, if the body is acted upon by forces such that their resultant does not pass through the centre of mass, a couple also acts on the body. In graphical solutions, it is possible to replace inertia force and inertia couple by an equivalent offset inertia force which can account for both. This is done by displacing the line of action of the inertia force from the centre of mass. The perpendicular displacement h of the force from the centre of mass is such that the torque so produced is equal to the inertia couple acting on the body,

i.e. $\quad T_i = C_i$

or $\quad F_i \times h = C_i$

or
$$h = \frac{C_i}{F_i} = \frac{-I_g\alpha}{-mf_g} = \frac{mk^2\alpha}{mf_g} = \frac{k^2\alpha}{f_g} \tag{13.5}$$

h is taken in such a way that the force produces a moment about the centre of mass, which is opposite in sense to the angular acceleration α.

13.3 DYNAMIC ANALYSIS OF FOUR-LINK MECHANISMS

For dynamic analysis of four-link mechanisms, the following procedure may be adopted:

1. Draw the velocity and acceleration diagrams of the mechanism from the configuration diagram by usual methods.
2. Determine the linear acceleration of the centres of masses of various links, and also the angular accelerations of the links.
3. Calculate the inertia forces and inertia couples from the relations $\mathbf{F}_i = -m\mathbf{f}_g$ and $\mathbf{C}_i = -I_g\alpha$.
4. Replace $\mathbf{F}_i$ with equivalent offset inertia force to take into account $\mathbf{F}_i$ as well as $\mathbf{C}_i$.
5. Assume equivalent offset inertia forces on the links as static forces and analyse the mechanism by any of the methods outlined in Chapter 12.

Example 13.1 *The dimensions of a four-link mechanism are AB = 500 mm, BC = 660 mm, CD = 560 mm and AD = 1000 mm.*

The link AB has an angular velocity of 10.5 rad/s counter-clockwise and an angular retardation of 26 rad/s² at the instant when it makes an angle of 60° with AD, the fixed link.

The mass of the links BC and CD is 4.2 kg/m length. The link AB has a mass of 3.54 kg, the centre of which lies at 200 mm from A and a moment of inertia of 88 500 kg.mm².

Neglecting gravity and friction effects, determine the instantaneous value of the drive torque required to be applied on AB to overcome the inertia forces.

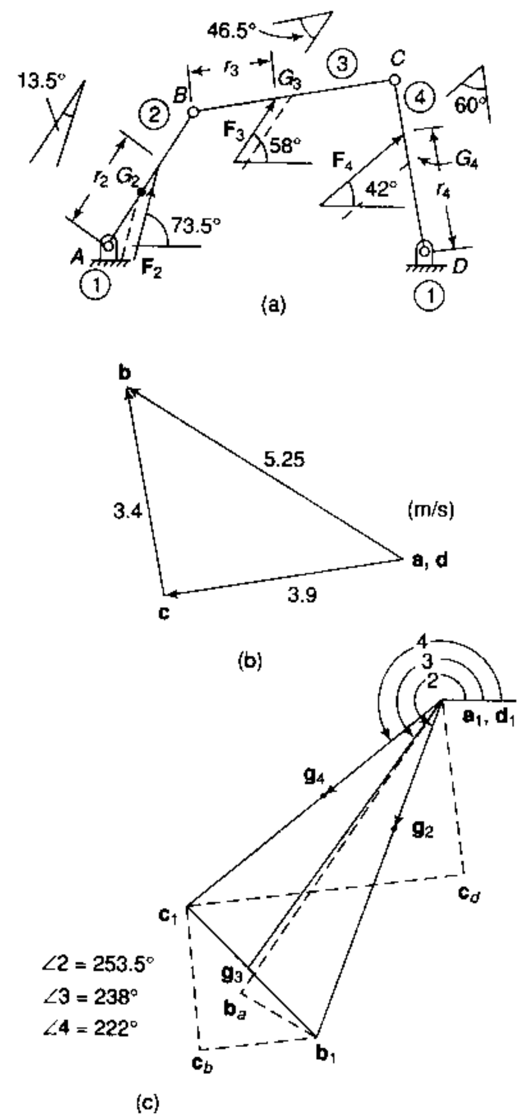


Fig. 13.1

Solution Draw the configuration diagram $ABCD$ of the mechanism to a suitable scale [Fig.13.1(a)]. The velocity and acceleration diagrams of the same have been shown in Figs 13.1 (b) and (c).

From the velocity diagram,

v_b or $ab = \omega_{ba} \times AB = 10.5 \times 0.5 = 5.25$ m/s

v_{cd} or $bc = 3.4$ m/s and v_c or $dc = 3.9$ m/s

From the acceleration diagram,

$$f_{ba}^c = \frac{(ab)^2}{AB} = \frac{(5.25)^2}{0.5} = 55.1 \text{ m/s}^2$$

$$f_{ba}^t = \alpha \times AB = 26 \times 0.5 = 13 \text{ m/s}^2$$

$$f_{cb}^c = \frac{(bc)^2}{BC} = \frac{(3.4)^2}{0.66} = 17.5 \text{ m/s}^2$$

$$f_{cd}^c = \frac{(dc)^2}{DC} = \frac{(3.9)^2}{0.56} = 27.2 \text{ m/s}^2$$

Mass of the links

$m_2 = 3.54$ kg

$m_3 = 0.66 \times 4.2 = 2.77$ kg

$m_4 = 0.56 \times 4.2 = 2.35$ kg

Let G_2, G_3 and G_4 denote the centres of masses of links AB, BC and CD respectively. G_2 lies at 200 mm from A, and G_3 and G_4 at the midpoints of BC and CD respectively. Locate these points in the acceleration diagram. Measure the accelerations of G_2, G_3 and G_4.

$F_{g2} = 22.6$ m/s^2 ∠253.5°

$F_{g3} = 52.0$ m/s^2 ∠238°

$F_{g4} = 25.7$ m/s^2 ∠222°

Now find the inertia on the links. These act through their respective centres of mass in the directions opposite to that of accelerations.

$\mathbf{F}_2 = m_2 f_{g2} = 80$ N ∠73.5° (253.5° – 180°)

$\mathbf{F}_3 = m_3 f_{g3} = 144$ N ∠58° (238° – 180°)

$\mathbf{F}_4 = m_4 f_{g4} = 60$ N ∠42° (222° – 180°)

To determine the inertia couples, angular accelerations of the links are to be found.

$\alpha_2 = 26$ rad/s^2 clockwise

$$\alpha_3 = \frac{f_{cb}^t}{CB} = \frac{22.5}{0.66} = 34.1 \text{ rad/s}^2 \text{counter-clockwise}$$

$$\alpha_4 = \frac{f_{cd}^t}{CD} = \frac{44.3}{0.56} = 79.1 \text{ rad/s}^2 \text{counter-clockwise}$$

Then $C_i = I_g \alpha$

However, the inertia couples can be taken into account by replacing the inertia forces with equivalent offset inertia forces.

Now,

$$k_2^2 = \frac{I_g}{m_2} = \frac{88\ 500}{3.54} = 25\ 000 \text{ mm}^2$$

Links 3 and 4 have uniform cross sections,

$$k_3^2 = \frac{l^2}{12} = \frac{(660)^2}{12} = 36\ 300 \text{ mm}^2$$

$$k_4^2 = \frac{l^2}{12} = \frac{(560)^2}{12} = 26\ 133 \text{ mm}^2$$

and $$h_2 = \frac{k^2\alpha}{f_{g2}} = \frac{25\ 000 \times 26}{22\ 600} = 28.8 \text{ mm}$$

$$h_3 = \frac{36\,300 \times 34.1}{52\,000} = 23.8 \text{ mm}$$

$$h_4 = \frac{26\,133 \times 79.1}{25\,700} = 80.4 \text{ mm}$$

Also,

$$r_2 = 200 + \frac{28.8}{\sin 13.5°} = 325 \text{ mm}$$

$$r_3 = 330 - \frac{23.8}{\sin 46.5°} = 297 \text{ mm}$$

$$r_4 = 280 + \frac{80.3}{\sin 60°} = 373 \text{ mm}$$

An inertia couple acts in a direction opposite to that of the angular acceleration. Thus, offsets h_2, h_3 and h_4 are to be such that the required inertia couples are set up. For example, the angular acceleration of the link 2 is clockwise (being retardation). Therefore, inertia couple must be counter-clockwise. Links 2 and 3 have counter-clockwise accelerations and Thus, the inertia couples are to be clockwise.

Now, assume equivalent offset inertia forces on the links as static forces and solve. This has been done in Examples 12.9 and 12.12.

The required input torque 23.5 N.m (counter-clockwise)

13.4 DYNAMIC ANALYSIS OF SLIDER-CRANK MECHANISMS

The steps outlined for dynamic analysis of a four-link mechanism also hold good for a slider-crank mechanism and the analysis can be carried out in exactly the same manner.

However, an analytical approach is also being described in detail in the following sections.

13.5 VELOCITY AND ACCELERATION OF A PISTON

Figure 13.2 shows a slider-crank mechanism in which the crank OA rotates in the clockwise direction. l and r are the lengths of the connecting rod and the crank respectively.

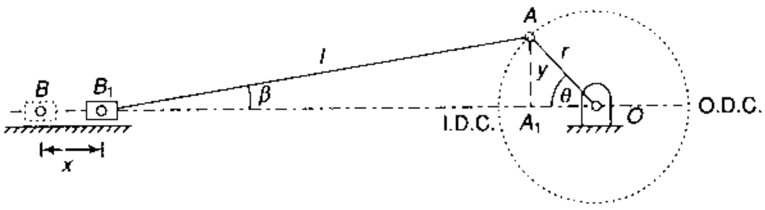


Fig. 13.2

Let x = displacement of piston from inner-dead centre

At the moment when the crank has turned through angle θ from the inner-dead centre,

$$x = B_1B = BO - B_1O$$
$$= BO - (B_1A_1 + A_1O)$$
$$= (l + r) - (l \cos \beta + \cos \theta)$$
$$= (nr + r) - (nr \cos \beta + r \cos \theta) \quad \text{(taking } l/r = n\text{)}$$
$$= r\,[(n + 1) - (n \cos \beta + \cos \theta)] \quad (13.6)$$

where $$\cos \beta = \sqrt{1 - \sin^2 \beta}$$
$$= \sqrt{1 - \frac{y^2}{l^2}}$$

$$= \sqrt{1 - \frac{(r\sin\theta)^2}{l^2}}$$

$$= \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$x = r[(n+1) - (\sqrt{n^2 - \sin^2\theta} + \cos\theta)]$$

$$= r[(1 - \cos\theta) + (n - \sqrt{n^2 - \sin^2\theta})] \tag{13.7}$$

If the connecting rod is very large as compared to the crank, n^2 will be large and the maximum value of $\sin^2\theta$ can be unity. Then $\sqrt{n^2 - \sin^2\theta}$ will be approaching $\sqrt{n^2}$ or n, and

$$x = r(1 - \cos\theta) \tag{13.8}$$

This is the expression for a simple harmonic motion. Thus, the piston executes a simple harmonic motion when the connecting rod is large.

Velocity of Piston

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}[r\{(1 - \cos\theta) + n - (n^2 - \sin^2\theta)^{1/2}\}]\frac{d\theta}{dt}$$

$$= r[(0 + \sin\theta) + 0 - \frac{1}{2}(n^2 - \sin^2\theta)^{1/2}(-2\sin\theta\cos\theta)]\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] \tag{13.9}$$

If n^2 is large compared to $\sin^2\theta$,

$$v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right] \tag{13.10}$$

If $\frac{\sin 2\theta}{2n}$ can be neglected (when n is quite large),

$$v = r\omega\sin\theta \tag{13.11}$$

Acceleration of Piston

$$f = \frac{dv}{dt} = \frac{dv d\theta}{d\theta dt}$$

$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]\omega$$

$$= r\omega\left(\cos\theta + \frac{2\cos 2\theta}{2n}\right)\omega$$

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \qquad (13.12)$$

If n is very very large,

$$f = r\omega^2 \cos\theta \text{ as in case of SHM} \qquad (13.13)$$

When $\theta = 0°$, i.e., at *IDC*, $f = r\omega^2\left(1 + \frac{1}{n}\right)$

When $\theta = 180°$, i.e., at *ODC*, $f = r\omega^2\left(-1 + \frac{1}{n}\right)$

At $\theta = 180°$, when the direction of motion is reversed,

$$f = r\omega^2\left(1 - \frac{1}{n}\right) \qquad (13.14)$$

Note that this expression of acceleration has been obtained by differentiating the approximate expression for the velocity. It is, usually, very cumbersome to differentiate the exact expression for velocity. However, this gives satisfactory results.

13.6 ANGULAR VELOCITY AND ANGULAR ACCELERATION OF CONNECTING ROD

As $y = l\sin\beta = r\sin\theta$

$\therefore$

$$\sin\beta = \frac{\sin\theta}{n} \qquad (n = l/r)$$

Differentiating with respect to time,

$$\cos\beta\frac{d\beta}{dt} = \frac{1}{n}\cos\theta\frac{d\theta}{dt}$$

$$\frac{d\beta}{dt} = \frac{\cos\theta}{n\cos\beta}\omega$$

or

$$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \qquad (13.15)$$

where ω_c is the angular velocity of the connecting rod

$$= \omega\frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Let α_c = angular acceleration of the connecting rod

$$= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}[\cos\theta\,(n^2 - \sin^2\theta)^{-1/2}]\omega$$

$$= \omega^2 [-\cos\theta \frac{1}{2}(n^2 - \sin^2\theta)^{-3/2}(-2\sin\theta\cos\theta) + (n^2 - \sin^2\theta)^{-1/2}(-\sin\theta)]$$

$$= \omega^2 \sin\theta \left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$= -\omega^2 \sin\theta \left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right] \quad (13.16)$$

The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β. Thus, in the given case, the angular acceleration of the connecting rod is clockwise.

13.7 ENGINE FORCE ANALYSIS

An engine is acted upon by various forces such as weight of reciprocating masses and connecting rod, gas forces, forces due to friction and inertia forces due to acceleration and retardation of engine elements, the last being dynamic in nature. In this section, the analysis is made of the forces neglecting the effect of the weight and the inertia effect of the connecting rod.

(i) Piston Effort (Effective Driving Force)

The piston effort is termed as the net or effective force applied on the piston. In reciprocating engines, the reciprocating masses accelerate during the first half of the stroke and the inertia force tends to resist the same. Thus, the net force on the piston is decreased. During the later half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of the applied gas pressure and thus, increases the effective force on the piston.

In a vertical engine, the weight of the reciprocating masses assists the piston during the outstroke (down stroke), thus, increasing the piston effort by an amount equal to the weight of the piston. During the instroke (upstroke), the piston effort is decreased by the same amount.

Let A_1 = area of the cover end
A_2 = area of the piston rod end
P_1 = pressure on the cover end
P_2 = pressure on the rod end
m = mass of the reciprocating parts

Force on the piston due to gas pressure, $F_p = p_1 A_1 - p_2 A_2$ (13.17)

Inertia force, $F_b = mf = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$ (13.18)

which is in the opposite direction to that of the acceleration of the piston.

Net (effective) force on the piston, $F = F_p - F_b$ (13.19)

In case friction resistance F_f is also taken into account,

Force on the piston, $F = F_p - F_b - F_f$

In case of vertical engines, the weight of the piston or reciprocating parts also acts as force and thus force on the piston, $F = F_p + mg - F_b - F_f$

(ii) Force (thrust) along the Connecting Rod

Let F_c = Force in the connecting rod (Fig. 13.3)

Then equating the horizontal components of forces,

$$F_c \times \cos\beta = F \quad \text{or} \quad F_c = \frac{F}{\cos\beta}$$

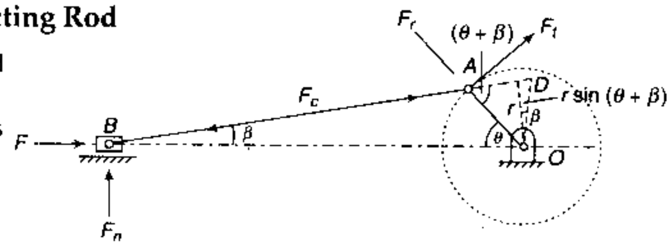


Fig. 13.3

(iii) Thrust on the Sides of Cylinder

It is the normal reaction on the cylinder walls.

$$F_n = F_c \sin\beta = F \tan\beta$$

(iv) Crank Effort

Force is exerted on the crankpin as a result of the force on the piston. *Crank effort* is the net effort (force) applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft.

Let F_t = crank effort

As

$$F_t \times r = F_c\, r \sin(\theta + \beta) \qquad \text{(refer to Fig. 13.3)}$$

$$\therefore \quad F_t = F_c \sin(\theta + \beta)$$

$$= \frac{F}{\cos\beta} \sin(\theta + \beta) \qquad (13.20)$$

(v) Thrust on the Bearings

The component of F_c along the crank (in the radial direction) produces a thrust on the crankshaft bearings.

$$F_r = F_c \cos(\theta + \beta) = \frac{F}{\cos\beta} \cos(\theta + \beta)$$

13.7 TURNING MOMENT ON CRANKSHAFT

$$T = F_t \times r$$

$$= \frac{F}{\cos\beta} \sin(\theta + \beta) \times r$$

$$= \frac{Fr}{\cos\beta} (\sin\theta\cos\beta + \cos\theta\sin\beta)$$

$$= Fr\left(\sin\theta + \cos\theta\sin\beta\frac{1}{\cos\beta}\right)$$

$$= Fr\left(\sin\theta + \cos\theta\,\frac{\sin\theta}{n}\,\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{2\sin\theta\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right) \qquad (13.21)$$

Also, as $r\sin(\theta+\beta) = OD\cos\beta$

$$T = F_t \times r$$

$$= \frac{F}{\cos\beta} r\sin(\theta+\beta) \qquad \text{[From (13.20)]}$$

$$= \frac{F}{\cos\beta}(OD\cos\beta)$$

$$= F \times OD \qquad (13.22)$$

Example 13.2 *A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m² and 60 kN/m² respectively. Diameter of the piston rod is 40 mm. Determine*

(i) turning moment on the crank shaft
(ii) thrust on the bearings
(iii) acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW

Solution

$r = 0.44/2 = 0.22$ m $l = 0.924$ m
$N = 210$ rpm $m = 20$ kg
$\theta = 30°$

$$n = l/r = 0.924/0.22 = 4.2$$

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 30°}{4.2} = 0.119$$

or $\beta = 6.837°$

$$F_p = (p_1A_1 - p_2A_2)$$

$$= (500 \times 10^3 \times \frac{\pi}{4} \times 0.22^2$$
$$-60 \times 10^3 \times \frac{\pi}{4} \times (0.22^2 - 0.04^2))$$

$$= 19\,007 - 2206$$
$$= 16\,801 \text{ N}$$

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 20 \times 0.22 \times (22)^2\left(\cos 30° + \frac{\cos 60°}{4.2}\right)$$

$$= 2098 \text{ N}$$

Piston effort, $F = F_p - F_b$

$$= 16\,801 - 2098 = 14\,703 \text{ N}$$

(i) Turning moment, $T = \frac{F}{\cos\beta}\sin(\theta+\beta) \times r$

$$= \frac{14\,703}{\cos 6.837}\sin(30° + 6.837°) \times 0.22$$

$$= 1953 \text{ N.m}$$

(ii) Thrust on the bearings, F_r

$$= \frac{F}{\cos\beta}\cos(\theta+\beta)$$

$$= \frac{14\ 703}{\cos 6.837} \cos(30^\circ + 6.837^\circ)$$

$= 11\ 852$ N

(iii) Accelerating torque = Turning moment − Resisting torque

Resisting torque can be found from

$P = T\omega$

or $22\ 000 = T \times 22$

or $T = 1000$ N.m

∴ Accelerating torque $= 1953 - 1000$

or $I\alpha = mk^2.\ \alpha = 953$

or $8 \times 0.6^2 \times \alpha = 953$

or Acceleration of flywheel, $\alpha = 330.9$ rad/s^2

Example 13.3 ***The crank and connecting rod of a vertical petrol engine, running at 1800 rpm are 60 mm and 270 mm respectively. The diameter of the piston is 100 mm and the mass of the reciprocating parts is 1.2 kg. During the expansion stroke when the crank has turned 20° from the top dead centre, the gas pressure is 650 kN/m². Determine the***

(i) net force on the piston

(ii) net load on the gudgeon pin

(iii) thrust on the cylinder walls

(iv) speed at which the gudgeon pin load is reversed in direction

Solution

$r = 0.06$ m	$l = 0.27$ m
$N = 1800$ rpm	$p = 650$ kN/m^2
$m = 1.2$ kg	$d = 0.1$ m
$\theta = 20^\circ$	

Refer Fig. 13.4,

$n = l/r = 0.27/0.06 = 4.5$

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

$$\cos\beta = \frac{1}{n}\sqrt{n^2 - \sin^2\theta} = \frac{1}{4.5}\sqrt{4.5^2 - \sin^2 20^\circ} = 0.9971$$

∴ $\beta = 4.36^\circ$

$$\left(\text{or} \quad \sin\beta = \frac{\sin\theta}{n} = \frac{\sin 20^\circ}{4.5} = 0.076 \text{ or } \beta = 4.36^\circ\right)$$

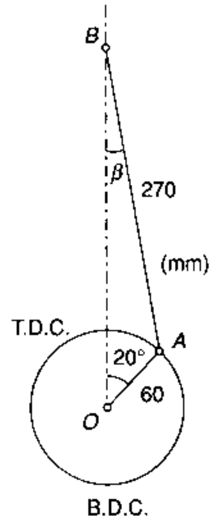


Fig. 13.4

Force due to gas pressure, F_p = Area × Pressure

$$= \frac{\pi}{4}(d)^2 \times p = \frac{\pi}{4}(0.1)^2 \times 650 \times 10^3$$

$= 5105$ N

$$\text{Inertia force, } F_b = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 1.2 \times 0.06 \times (188.5)^2\left(\cos 20^\circ + \frac{\cos 40^\circ}{4.5}\right)$$

$= 2840$ N

(i) Net (effective) force on the piston,

$F = F_p - F_b + mg$

$= 5105 - 2840 + 1.2 \times 9.81$

$= \underline{2276.8}$ N

(ii) Net load on the gudgeon pin = Force in the connecting rod

$$= \frac{F}{\cos\beta} = \frac{2276.8}{0.9971} = \underline{2283.4} \text{ N}$$

(iii) Thrust on the cylinder walls $= F\tan\beta$

$= 2276.8 \tan 4.36^\circ = 173.5$ N

(iv) Speed at which the gudgeon pin load is reversed in direction,

$$F = F_p - mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + mg$$

$$0 = 5105 - 1.2 \times 0.06\omega^2\left(\cos 20^\circ + \frac{\cos 40^\circ}{4.5}\right) + 1.2 \times 9.81$$

$$0.079\,91\,\omega^2 = 5116.8$$

$$\omega = 253.04$$

$$\frac{2\pi N}{60} = 253.04$$

$$N = \underline{2416.3} \text{ rpm}$$

Example 13.4 *In a vertical double-acting steam engine, the connecting rod is 4.5 times the crank. The weight of the reciprocating parts is 120 kg and the stroke of the piston is 440 mm. The engine runs at 250 rpm. If the net load on the piston due to steam pressure is 25 kN when the crank has turned through an angle of 120° from the top dead centre, determine the*

(i) thrust in the connecting rod
(ii) pressure on slide bars
(iii) tangential force on the crank pin
(iv) thrust on the bearings
(v) turning moment on the crankshaft.

Solution

$r = 0.44/2 = 0.22$ m $\quad N = 250$ rpm
$F = 25$ kN $\quad m = 120$ kg
$\theta = 120°$ $\quad n = 4.5$

$$\omega = \frac{2\pi \times 250}{60} = 26.18 \text{ rad/s}$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 120^\circ}{4.5} = 0.1925$$

or $\quad \beta = 11.1°$

Accelerating force, $F_b = mr\omega^2\left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

$$= 120 \times 0.22 \times (26.18)^2\left(\cos 120^\circ + \frac{\cos(240^\circ)}{4.5}\right)$$

$= -11\,058$ N

Force on the piston, $F = F_p + mg - F_b$
$= 25\,000 + 120 \times 9.81 - (-11\,058)$
$= 37\,235$ N

(i) Thrust in the connecting rod,

$$F_c = \frac{F}{\cos\beta} = \frac{37\,235}{\cos 11.1} = 37\,945 \text{ N}$$

(ii) Pressure on slide bars,
$F_n = F\tan\beta = 37\,235 \tan 11.1° = 7305$ N

(iii) Tangential force on the crank pin
$F_t = F_c \sin(\theta + \beta)$
$= 37\,945 \times \sin(120° + 11.1°) = 28\,594$ N

(iv) Thrust on the bearings,
$F_r = F_c \cos(\theta + \beta) = 37\,945 \times \cos(120° + 11.1°) = -24\,944$ N

(v) Turning moment on the crankshaft
$T = F_t \times r = 28\,594 \times 0.22 = 6290.7$ N.m

Example 13.5 *The crank and the connecting rod of a vertical single cylinder gas engine running at 1800 rpm are 60 mm and 240 mm respectively. The diameter of the piston is 80 mm and the mass of the reciprocating parts is 1.2 kg. At a point during the power stroke when the piston has moved 20 mm from the top dead centre position, the pressure on the piston is 800 kN/m². Determine the*

(i) net force on the piston
(ii) thrust in the connecting rod
(iii) thrust on the sides of cylinder walls
(iv) engine speed at which the above values are zero.

Solution

$r = 0.06$ m $\quad l = 0.24$ m
$N = 1800$ rpm $\quad m = 1.2$ kg
$n = 0.24/0.06 = 4$ $\quad d = 0.08$ m

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

Draw the configuration for the given position to some scale (Fig. 13.5) and obtain angle θ which is found to be 43.5°.

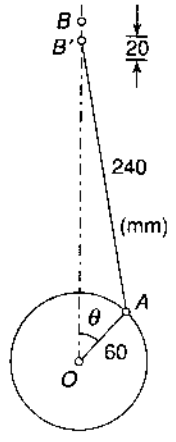


Fig. 13.5

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 43.5^\circ}{4} = 0.1721$$

or $\beta = 9.91^\circ$

Force due to gas pressure,

F_p = Area × Pressure

$$= \frac{\pi}{4}(d)^2 \times p = \frac{\pi}{4}(0.08)^2 \times 800 \times 10^3 = 4021 \text{ N}$$

Accelerating force, $F_b = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 1.2 \times 0.06 \times (188.5)^2\left(\cos 43.5^\circ + \frac{\cos 87^\circ}{4}\right)$$

= 1889 N

(i) Force on the piston, $F = F_p + mg - F_b$
= 4021 + 1.2 × 9.81 − 1889
= 2144 N

(ii) Thrust in the connecting rod,

$$F_c = \frac{F}{\cos\beta} = \frac{2144}{\cos 9.91^\circ} = 2176 \text{ N}$$

(iii) Thrust on the sides of cylinder walls,
$F_n = F\tan\beta = 2176 \tan 9.91^\circ = 380$ N

(iv) The above values are zero at the speed when the force on the piston F is zero.

$$F = F_p - mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + mg$$

$$0 = 4021 - 1.2 \times 0.06\,\omega^2\left(\cos 43.5^\circ + \frac{\cos 87^\circ}{4}\right) + 1.2 \times 9.81$$

$$0.053\,17\,\omega^2 = 4032.8$$

$$\omega = 75\,849$$

$$\frac{2\pi N}{60} = 275.4$$

$$N = 2630 \text{ rpm}$$

13.8 DYNAMICALLY EQUIVALENT SYSTEM

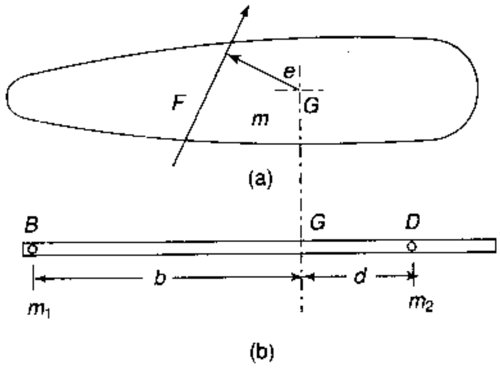


Fig. 13.6

In the previous section, the expression for the turning moment of the crankshaft has been obtained for the net force F on the piston. This force F may be the gas force with or without the consideration of inertia force acting on the piston. As the mass of the connecting rod is also significant, the inertia due to the same should also be taken into account. As neither the mass of the connecting rod is uniformly distributed nor the motion is linear, its inertia cannot be found as such. Usually, the inertia of the connecting rod is taken into account by considering a *dynamically-equivalent system*. A dynamically equivalent system means that the rigid link is replaced by a link with two point masses in such a way that it has the same motion as the rigid link when subjected to the same force, i.e., the centre of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.

Figure 13.6 (a) shows a rigid body of mass m with the centre of mass at G. Let it be acted upon by a force F which produces linear acceleration f of the centre of mass as well as the angular acceleration of the body as the force F does not pass through G.

As we know, $F = m.f$ and $F.e = I.\,\alpha$

Acceleration of G, $$f = \frac{F}{m}$$

Angular acceleration of the body, $\alpha = \frac{F.e}{I}$
where e = perpendicular distance of F from G
and I = moment of inertia of the body about perpendicular axis through G

Now to have the dynamically equivalent system, let the replaced massless link [Fig. 13.6(b)] has two point masses m_1 (at B and m_2 at D) at distances b and d respectively from the centre of mass G as shown in Fig. 13.6 (b).

1. To satisfy the first condition, as the force F is to be same, the sum of the equivalent masses m_1 and m_2 has to be equal to m to have the same acceleration. Thus, $m = m_1 + m_2$.
2. To satisfy the second condition, the numerator $F.e$ and the denominator I must remain the same. F is already taken same, Thus, e has to be same which means that the perpendicular distance of F from G should remain same or the combined centre of mass of the equivalent system remains at G. This is possible if

$$m_1 b = m_2 d$$

To have the same moment of inertia of the equivalent system about perpendicular axis through their combined centre of mass G, we must have

$$I = m_1 b^2 + m_2 d^2$$

Thus, any distributed mass can be replaced by two point masses to have the same dynamical properties if the following conditions are fulfilled:

(i) The sum of the two masses is equal to the total mass.
(ii) The combined centre of mass coincides with that of the rod.
(iii) The moment of inertia of two point masses about the perpendicular axis through their combined centre of mass is equal to that of the rod.

13.9 INERTIA OF THE CONNECTING ROD

Let the connecting rod be replaced by an equivalent massless link with two point masses as shown in Fig. 13.7. Let m be the total mass of the connecting rod and one of the masses be located at the small end B. Let the second mass be placed at D and

m_b = mass at B

m_d = mass at D

Take, $BG = b$ and $DG = d$
Then

$$m_b + m_d = m \quad \text{(i)}$$

and

$$m_b . b = m_d . d \quad \text{(ii)}$$

From (i) and (ii)

$$m_b + \left(m_b \frac{b}{d}\right) = m$$

or

$$m_b \left(1 + \frac{b}{d}\right) = m$$

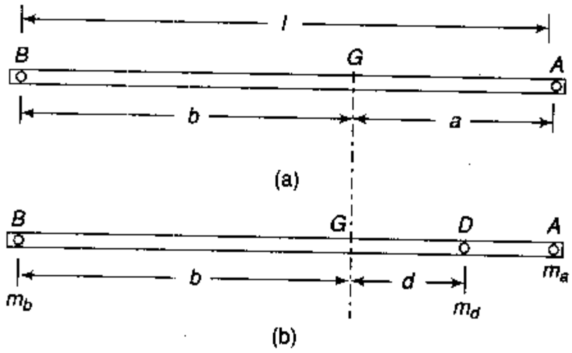


Fig. 13.7

or $$m_b\left(\frac{b+d}{d}\right)=m$$

or $$m_b=m\frac{d}{b+d}$$

Similarly,

$$m_d=m\frac{b}{b+d}$$

Also, $$I=m_b b^2+m_d d^2$$

$$=m\frac{d}{b+d}b^2+m\frac{b}{b+d}d^2$$

$$=mbd\left(\frac{b+d}{b+d}\right)$$

$$=mbd \qquad (13.23)$$

Let k = radius of gyration of the connecting rod about an axis through the centre of mass G perpendicular to the plane of motion.

Then $$mk^2=mbd$$

or $$k^2=bd \qquad (13.24)$$

This result can be compared with that of an equivalent length of a simple pendulum in the following manner:

The equivalent length of a simple pendulum is given by

$$L=\frac{k^2}{b}+b=d+b \qquad \left(\frac{k^2}{b}=d\right)$$

where b is the distance of the point of suspension from the centre of mass of the body and k is the radius of gyration. Thus, in the present case, $d+b$ (= L) is the equivalent length if the rod is suspended from the point B, and D is the centre of oscillation or percussion.

However, in the analysis of the connecting rod, it is much more convenient if the two point masses are considered to be located at the centre of the two end bearings, i.e., at A and B.

Let m_a = mass at A, distance $AG=a$

Then $m_a+m_b=m$

$$m_a=m\frac{b}{a+b}=m\frac{b}{l} \qquad (l = \text{length of rod})$$

$$m_b=m\frac{a}{a+b}=m\frac{a}{l}$$

$$I'=mab$$

Assuming $a>d$, $I'>I$

This means that by considering the two masses at A and B instead of at D and B, the inertia torque is increased from the actual value ($T = I\alpha_c$). The error is corrected by incorporating a correction couple.

Then,

$$
\begin{aligned}
\text{correction couple, } \Delta T &= \alpha_c\,(mab - mbd)\\
&= mb\alpha_c\,(a - d)\\
&= mb\alpha_c\,[(a + b) - (b + d)]\\
&= mb\alpha_c\,(l - L) \qquad \text{(taking } b + d = L\text{)} \qquad (13.25)
\end{aligned}
$$

This correction couple must be applied in the opposite direction to that of the applied inertia torque. As the direction of the applied inertia torque is always opposite to the direction of the angular acceleration, the direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β.

The correction couple will be produced by two equal, parallel and opposite forces F_y acting at the gudgen pin and crankpin ends perpendicular to the line of stroke (Fig. 13.8). The force at B is taken by the reaction of guides.

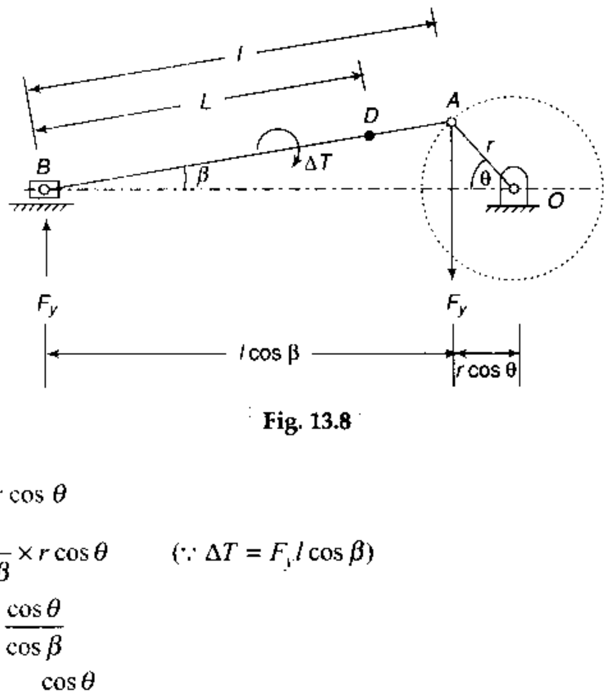


Fig. 13.8

Turning moment at crankshaft due to force at A or correction torque,

$$
\begin{aligned}
T_c &= F_y \times r\cos\theta\\
&= \frac{\Delta T}{l\cos\beta} \times r\cos\theta \qquad (\because \Delta T = F_y l\cos\beta)\\
&= \frac{\Delta T}{(l/r)}\,\frac{\cos\theta}{\cos\beta}\\
&= \Delta T\,\frac{\cos\theta}{n\,\frac{1}{n}\sqrt{n^2 - \sin^2\theta}}\\
&= \Delta T\,\frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}} \qquad (13.26)
\end{aligned}
$$

This correction torque is to be deducted from the inertia torque acting on the crankshaft.

Also, due to the weight of the mass at A, a torque is exerted on the crankshaft which is given by

$$T_a = (m_a g) r\cos\theta \qquad (13.27)$$

In case of vertical engines, a torque is also exerted on the crankshaft due to the weight of mass at B and the expression will be similar to Eq. (13.21), i.e.,

$$T_b = (m_b g)\,r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \qquad (13.28)$$

The net torque or turning moment on the crankshaft will be the algebraic sum of the

(i) turning moment due to the force of gas pressure (T)

(ii) inertia torque due to the inertia force at the piston as a result of inertia of the reciprocating mass including the mass of the portion of the connecting rod (T_b)

(iii) inertia torque due to the weight (force) of the mass at the crank pin which is the portion of the mass of the connecting rod taken at the crank pin (T_a).

(iv) inertia torque due to the correction couple (T_c)

(v) turning moment due to the weight (force) of the piston in case of vertical engines

Usually, it is convenient to combine the forces at the piston occurring in (ii) and (v).

13.10 INERTIA FORCE IN RECIPROCATING ENGINES (GRAPHICAL METHOD)

The inertia forces in reciprocating engines can be obtained graphically as follows (Fig. 13.9).

1. Draw the acceleration diagram by Klein's construction (refer Section 3.8). Remember that the acceleration diagram is turned through 180° from the actual diagram and therefore, the directions of accelerations are towards O [Fig.13.9(a)].
2. Replace the mass of the connecting rod by a dynamically equivalent system of two masses. If one mass is placed at B, the other will be at D given by $d = k^2/b$, where k is the radius of gyration and b and d are the distances of the centre of mass from B and D respectively.

   Point D can also be obtained graphically. Draw $GE \perp AB$ at G and take $GE = k$. Make $\angle BED = 90°$, and obtain the point D on AB.
3. Obtain the accelerations of points G and D from the acceleration diagram by locating the points g_1 and d_1 on Ab_1 which represents the total acceleration of the connecting rod.

   As Ad_1/AD and Ag_1/AG are equal to Ab_1/AB, Dd_1 and Gg_1 can be drawn parallel to OB. Thus, d_1O and g_1O represent accelerations of points D and G respectively.
4. The acceleration of the mass at B is along BO and in the direction B to O. Therefore, the inertia force due to this mass acts in the opposite direction.

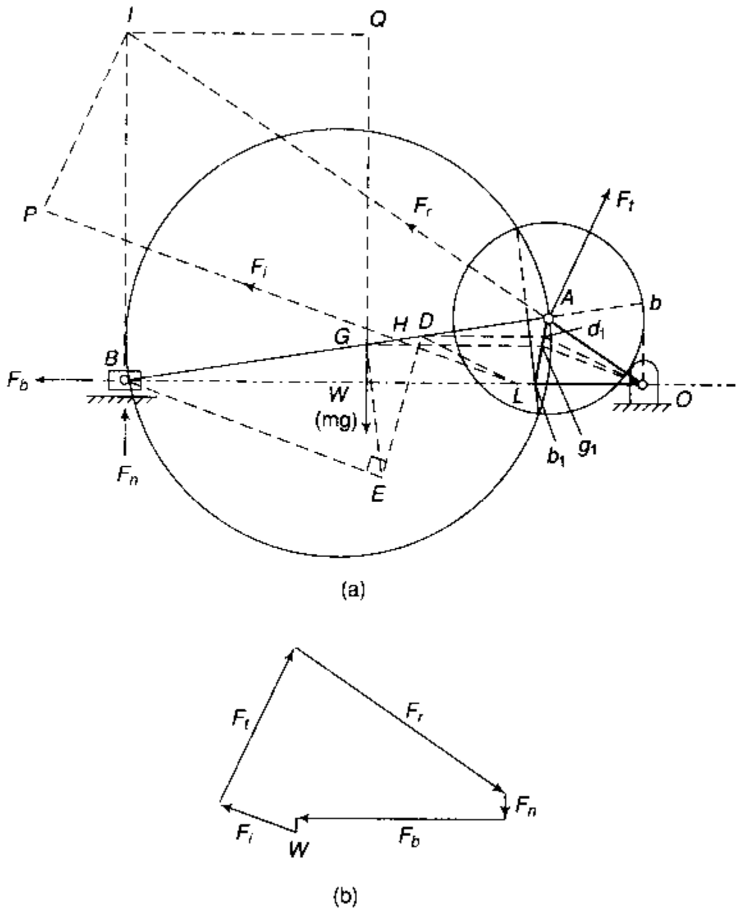


Fig. 13.9

5. The acceleration of the mass at D is parallel to d_1O and in the direction d_1 to O, therefore, the inertia force due to this mass acts in the opposite direction through D. Draw a line parallel to Od_1 through D to represent the direction of the inertia force.

Let the lines of action of the two inertia forces due to masses at B and D meet at L. Then the resultant of the forces which is the total inertia force of the connecting rod and is parallel to Og_1 must also pass through the point L. Therefore, draw a line parallel to Og_1 through L to represent the direction of the inertia force of the connecting rod.

Now, the connecting rod is under the action of the following forces:

- Inertia force of reciprocating part $\mathbf{F}_b$ along OB
- The reaction of the guide $\mathbf{F}_n$ (magnitude and direction sense unknown)
- Inertia force of the connecting rod $\mathbf{F}_i$
- The weight of the connecting rod $\mathbf{W}$ (= mg)
- Tangential force $\mathbf{F}_t$ at the crank pin (to be found)
- Radial force $\mathbf{F}_r$ at the crank pin along OA (magnitude and direction sense unknown).

Produce the lines of action of $\mathbf{F}_r$ and $\mathbf{F}_n$ to meet at I, the instantaneous centre of the connecting rod. Draw IP and IQ perpendicular to the lines of action of $\mathbf{F}_i$ and the weight $\mathbf{W}$ respectively.

For the equilibrium of the connecting rod, taking moments about I,

$$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ \tag{13.29}$$

Obtain the value of F_t from it and draw the force polygon to find the magnitudes and directions of forces $\mathbf{F}_r$ and $\mathbf{F}_n$ [Fig. 13.9(b)].

In the above equation, F_t is the force required for the static equilibrium of the mechanism or it is the force required at the crank pin to overcome the inertia of the reciprocating parts and of the connecting rod. If it indicates a clockwise torque, then

Inertia torque on the crankshaft = $F_t \times OA$ counter-clockwise

Example 13.6 *The following data relate to the connecting rod of a reciprocating engine:*

Mass	= *50 kg*
Distance between bearing centres	= *900 mm*
Diameter of big end bearing	= *100 mm*
Diameter of small end bearing	= *80 mm*

Time of oscillation when the connecting rod is suspended from

big end = 1.7 s

small end = 1.85 s

Determine the

(i) radius of gyration k of the rod about an axis through centre of mass perpendicular to the plane of oscillation,

(ii) moment of inertia of the rod about the same axis, and

(iii) dynamically equivalent system of the connecting rod comprising two masses, one at the small end-bearing centre.

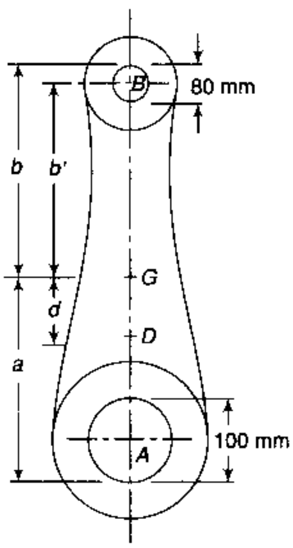


Fig. 13.10

Solution Refer Fig. 13.10.

Let L_a = length of equivalent simple pendulum when suspended from the top of the big end bearing

L_b = length of equivalent simple pendulum when suspended from the top of the small end bearing

a = distance of the centre of mass G from top of big-end bearing

b = distance of the centre of mass G from top of small-end bearing

$$t_a = 2\pi\sqrt{\frac{L_a}{g}} \quad \text{and} \quad t_b = 2\pi\sqrt{\frac{L_b}{g}}$$

or $$1.7 = 2\pi\sqrt{\frac{L_a}{g}} \quad \text{and} \quad 1.85 = 2\pi\sqrt{\frac{L_a}{9.81}}$$

or $L_a = 0.7181$ m and $L_b = 0.8505$ m

or $$a + \frac{k^2}{a} = 0.7181 \quad \text{and} \quad b + \frac{k^2}{b} = 0.8505$$

or $k^2 = 0.7181a - a^2 = 0.8505b - b^2$ (i)

But $a + b = 900 + \frac{100}{2} + \frac{80}{2} = 990 \text{ mm} = 0.99 \text{ m}$

$a = 0.99 - b$

$\therefore$ (i) becomes $0.7181(0.99 - b) - (0.99 - b)^2$
$= 0.8505b - b^2$

or $0.7109 - 0.7181b - (0.9801 + b^2 - 1.98b)$
$= 8505b - b^2$

or $0.4115b = 0.2692$

or $b = 0.654$ m

$a = 0.99 - 0.654 = 0.336$ m

$k^2 = 0.8505 \times 0.654 - (0.654)^2 = 0.1286$

or $k = \underline{0.358 \text{ m}}$

$MOI, I = mk^2 = 50 \times (0.358)^2 = 6.4 \text{ kg.m}^2$

The distance of centre of mass of the connecting rod from the centre of the small end bearing, $b' = 654 - (80/2) = 614$ mm

Let the second mass be placed at D.

Take $GD = d$ and m_d = mass at D

Then

$$d = \frac{k^2}{b'} = \frac{0.1285}{0.614} = 0.209 \text{ m}$$

$$m_d = \frac{m \times b'}{b' + d} = \frac{50 \times 0.614}{0.614 + 0.209} = 37.3 \text{ kg}$$

$$m_b' = 50 - 37.3 = \underline{12.7 \text{ kg}}$$

(m'_b = mass at the small end-bearing centre)

Example 13.7 *The following data relate to a horizontal reciprocating engine:*

Mass of reciprocating parts	*= 120 kg*
Crank length	*= 90 mm*
Engine speed	*= 600 rpm*
Connecting rod:	
Mass	*= 90 kg*
Length between centres	*= 450 mm*
Distance of centre of mass from big end centre	*= 180 mm*
Radius of gyration about an axis through centre of mass	*= 150 mm*

Find the magnitude and the direction of the inertia torque on the crankshaft when the crank has turned 30° from the inner-dead centre.

Solution It is required to find the inertia torque, or turning moment, on the crankshaft due to the inertia of the piston as well as of the connecting rod. This can be obtained by analytical or graphical methods.

Analytical Method

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.8 \text{ rad/s}$$

Divide the mass of the connecting rod into two parts (Refer Fig. 13.11).

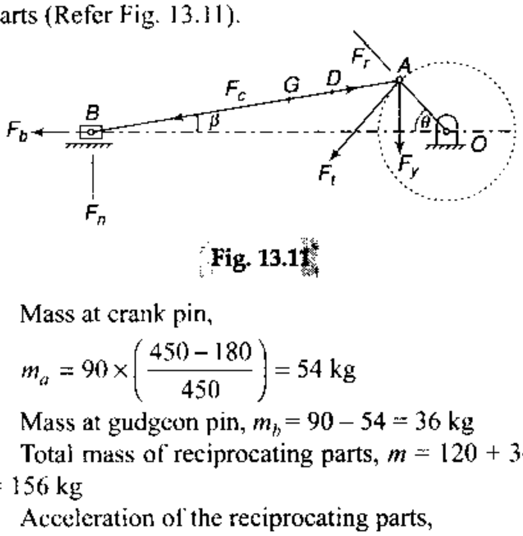


Fig. 13.11

Mass at crank pin,

$$m_a = 90 \times \left(\frac{450 - 180}{450}\right) = 54 \text{ kg}$$

Mass at gudgeon pin, $m_b = 90 - 54 = 36$ kg

Total mass of reciprocating parts, $m = 120 + 36 = 156$ kg

Acceleration of the reciprocating parts,

$$f = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

As θ is less than 90°, it is towards the right and Thus, the inertia force is towards left.

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$= 156 \times 0.09 \times (62.8)^2\left(\cos 30° + \frac{\cos 60°}{5}\right)$

$= 53\,490$ N

Inertia torque due to reciprocating parts

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \text{[Eq. (13.21)]}$$

$$= 53\,490 \times 0.09\left(\sin 30° + \frac{\sin 60°}{2\sqrt{(5)^2 - \sin^2 30°}}\right)$$

$= 2826$ N.m

(counter-clockwise as inertia force is towards left)

Correction couple due to assumed second mass of connecting rod at A,

$$\Delta T = m\alpha_c b(l - L) \quad \text{[Eq. (13.25)]}$$

where $b = 450 - 180 = 270$ mm

$l = 450$ mm

and $L = b + \frac{k^2}{b} = 270 + \frac{(150)^2}{270} = 353.3$ mm

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right] \quad \text{[Eq. (13.16)]}$$

$$= -(62.8)^2 \sin 30°\left[\frac{5^2 - 1}{(25 - \sin^2 30°)^{3/2}}\right]$$

$= -384.7$ rad/s^2

$\therefore \Delta T = 90 \times (-384.7) \times 0.27 \times (0.45 - 0.3533)$

$= -903.97$ N.m

The direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β as discussed in Section 13.9. Thus, it is clockwise.

$\therefore$ correction torque on the crankshaft,

$$T_c = \Delta T \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -903.97 \times \frac{\cos 30°}{\sqrt{25 - \sin^2 30°}}$$

$= -157.4$ N.m

Correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards left of the crankshaft, the correction torque is counter-clockwise.

Torque due to weight of mass at A,

$T_a = (m_a g)\, r \cos\theta$

$= 54 \times 9.81 \times 0.09 \times \cos 30°$

$= 41.3$ N.m counter-clockwise

$\therefore$ total inertia torque on the crankshaft

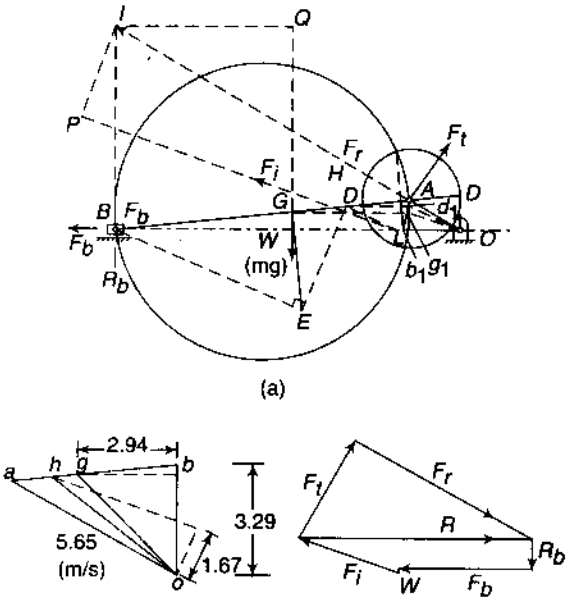
$= T_b - T_c + T_a$

$2826 - (-157.4) + 41.3$

$=$ <u>3024.7 N. m</u> counter-clockwise

Graphical Method

Draw the configuration diagram OAB of the engine mechanism to a convenient scale (Fig. 13.12) and its velocity and acceleration diagrams by Klein's construction (refer Section 13.10).



(b) (c)

Fig. 13.12

$v_a = \omega r = 62.8 \times 0.09 = 5.65$ m/s

$f_a = \omega^2 r = (62.8)^2 \times 0.09 = 355$ m/s^2

Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram represents the acceleration of A relative to O, i.e., 355 m/s², therefore, f_b can be obtained from

$$f_b = 355 \times \frac{\text{length } b_1O}{\text{length } OA}$$

It is found to be $f_b = 343.2$ m/s²

Similarly, $f_g = 345$ m/s²

$\therefore$ $F_b = m_b \times f_b = 120 \times 343.2 = 41186$ N

$F_i = m \times f_g = 90 \times 345 = 31050$ N

Complete the diagram of Fig. 13.12(a) as discussed in Section 13.10. Taking moments about I,

$$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$$

$$F_t \times 515 = 41\,186 \times 300 + 31\,050 \times 152 + 90 \times 9.81 \times 268$$

$$F_t = 33615.5 \text{ N.m}$$

$$\therefore \quad T = F_t \times r = 33615.5 \times 0.9 = \underline{3025.4 \text{ N.m}}$$

Instead of taking moments about the I-centre, the principle of virtual work can also be applied to obtain the torque as follows:

On the velocity diagram [Fig. 13.12(b)], locate the points b, h and g corresponding to B, H and G respectively and take the components of velocities in the directions of forces F_b, F_i and mg. In Klein's construction, the velocity diagram in turned through 90°. Then

$$T \times \omega = F_b \times v_b + F_i \times v_h + mg \times v_g$$

$$T \times 62.8 = 41\,186 \times 3.29 + 31\,050 \times 1.67 + 90 \times 9.81 \times 2.94$$

$$T = 2157.6 + 825.7 + 41.3$$

$$= \underline{3024.6 \text{ N.m}}$$

If it is desired to find the resultant force on the crank, complete the force diagram as shown in Fig. 13.12(c).

Resultant force on the crank pin, $R = \underline{70\,000}$ N at 0°

Example 13.8 *The connecting rod of a vertical reciprocating engine is 2 m long between centres and weighs 250 kg. The mass centre is 800 mm from the big end bearing. When suspended as a pendulum from the gudgeon pin axis, it makes 8 complete oscillations in 22 seconds. Calculate the radius of gyration of the rod about an axis through its mass centre. The crank is 400 mm long and rotates at 200 rpm. Find the inertia torque exerted on the crankshaft when the crank has turned through 40° from the top dead centre and the piston is moving downwards.*

Solution

Analytical method

Divide the mass of the rod into two parts (Fig. 13.13),

Mass at the crank pin,

$$m_a = 250 \times \frac{2.0 - 0.8}{2.0} = 150 \text{ kg}$$

Mass at the gudgeon pin,

$$m_b = 250 - 150 = 100 \text{ kg}$$

$$F = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= 100 \times 0.4 \times \left(\frac{2\pi \times 200}{60} \right)^2 \left(\cos 40° + \frac{\cos 80°}{2/0.4} \right)$$

$$= 100 \times 0.4 \times 438.6 \times 0.8$$

$$= 14\,049 \text{ N}$$

As it is a vertical engine, the weight (force) of the portion of the connecting rod at the piston pin also can be combined with this force, i.e.,

Net force = 14 049 − 100 × 9.81 = 13 068 N (upwards)

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

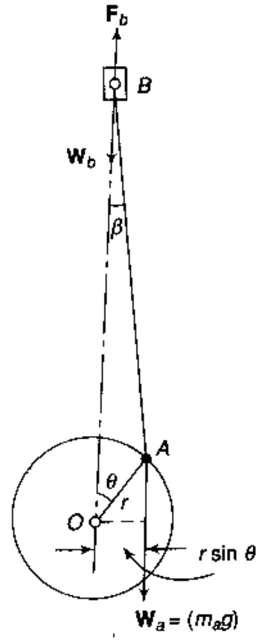


Fig. 13.13

$$= 13068 \times 0.4\left(\sin 40^\circ + \frac{\sin 80^\circ}{2\sqrt{25 - \sin^2 40^\circ}}\right)$$

$= 13\,068 \times 0.4 \times 0.7421$

$= 3879.1$ N.m counter-clockwise.

We have,

$$b + \frac{k^2}{b} = L$$

where $b = 2.0 - 0.8 = 1.2$ m and L can be found from

$$t = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad \frac{22}{8} = 2\pi\sqrt{\frac{L}{9.81}}$$

or $L = 1.88$ m

$\therefore$ $1.2 + \dfrac{k^2}{1.2} = 1.88$

or $k^2 = 0.816$

or $k = 0.903$

or radius of gyration = 903 mm

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$= -438.6 \sin 40^\circ\left[\frac{25 - 1}{(25 - \sin^2 40^\circ)^{3/2}}\right]$$

$= -55.5$ rad/s^2

$\Delta T = m\alpha_c b\,(l - L)$

$= 250 \times (-55.5) \times 1.2 \times (2.0 - 1.88)$

$= -1998$ N.m

The direction of the correction couple will be in the direction of decreasing angle β as discussed earlier. Thus, it is clockwise.

The correction torque on the crankshaft,

$$T_c = \Delta T\frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -1998 \times \frac{\cos 40^\circ}{\sqrt{25 - \sin^2 40^\circ}}$$

$= -308.7$ N.m

Correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards left on the upper side of crankshaft, the correction torque is counter-clockwise.

Torque due to weight of mass at A,

$T_a = m_a g\, r \sin\theta$

$= 150 \times 9.81 \times 0.4 \sin 40^\circ$

$= 378.3$ N.m clockwise

Total inertia torque on crankshaft $= T_b - T_c + T_a$

$= 3879.1 - (-308.7) - 378.3$

$= \underline{3809.5 \text{ N.m}}$

Graphical Method

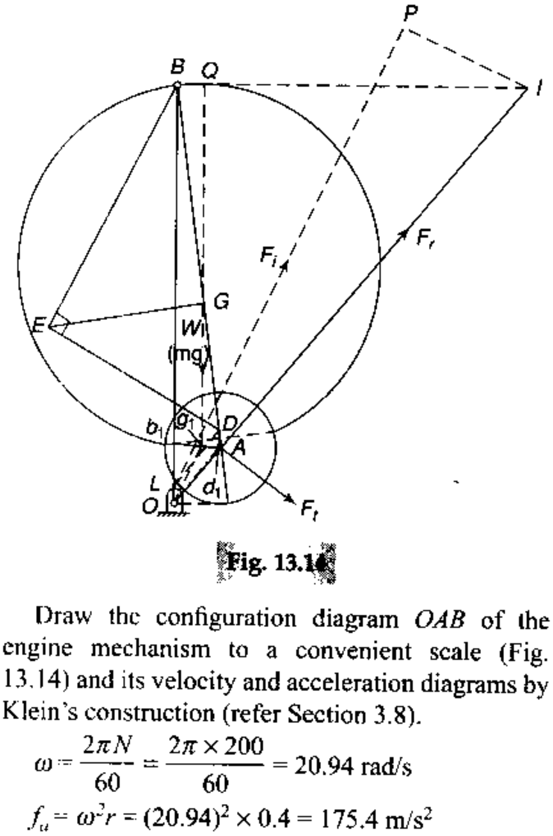


Fig. 13.14

Draw the configuration diagram OAB of the engine mechanism to a convenient scale (Fig. 13.14) and its velocity and acceleration diagrams by Klein's construction (refer Section 3.8).

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

$f_a = \omega^2 r = (20.94)^2 \times 0.4 = 175.4$ m/s^2

Complete the diagram of Fig. 13.14 as discussed in Section 13.10. Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram represents the acceleration of A relative to O, i.e., 175.4 m/s^2, therefore, f_b can be obtained from

$$f_b = 175.4 \times \frac{\text{length } b_1O}{\text{length } OA}$$

It is found to be $f_b = 143.8$ m/s^2

Similarly, $f_g = 153.4$ m/s^2

However, in this problem as the mass of the reciprocating parts is not given, inertia force due to the same is not to be calculated and Thus, f_b is not required.

Now, $F_i = m \times f_g = 250 \times 153.4 = 38\,350$ N

Taking moments about *I*,

$$F_t \times IA = F_i \times IP - mg \times IQ$$

$$F_t \times 260.9 = 38\,350 \times 76.2 - 250 \times 9.81 \times 176$$

$$F_t = 9546 \text{ N.m}$$

$\therefore \quad T = F_t \times r = 9546 \times 0.4 = \underline{3818 \text{ N.m}}$

$\therefore$

Example 13.9 ***For Example 13.8, determine the turning moment on the crankshaft if the bore of the cylinder is 700 mm and the gas pressure is 600 kN/m². Also, consider the mass of the piston which is 120 kg.***

Solution Total reciprocating mass at *B* = 100 + 120 = 220 kg

Force due to reciprocating mass,

$$F = 220 \times 0.4 \times \left(\frac{2\pi \times 200}{60}\right)^2 \left(\cos 40° + \frac{\cos 80°}{2/0.4}\right)$$

$= 220 \times 0.4 \times 438.6 \times 0.8$

$= 30\,877$ N

Net force on the piston = 30 877 – 100 × 9.81 = 29 896 N

Inertia torque

$$= 29\,896 \times 0.4\left(\sin 40° + \frac{\sin 80°}{2\sqrt{25 - \sin^2 40°}}\right)$$

$= 29\,896 \times 0.4 \times 0.7421$

$= 8874.3$ N.m

Net inertia torque on crankshaft

$= 8874.3 - (-308.7) - 378.3 = 8804.7 \text{ N.m} = 8.8047$

Now area of the cylinder bore $= \dfrac{\pi}{4}(0.7)^2$

$= 0.384\,85 \text{ m}^2$

Gas force = 0.384 85 × 600 = 230.9 kN

Turning moment = 230.9 × 0.4 × 0.7421

= 68.54 N.m

Therefore, turning moment available at the crank shaft = 68.54 – 8.8047

= $\underline{59.735 \text{ kN.m}}$

Example 13.10 ***The piston diameter of an internal combustion engine is 125 mm and the stroke is 220 mm. The connecting rod is 4.5 times the crank length and has a mass of 50 kg. The mass of the reciprocating parts is 30 kg. The centre of mass of the connecting rod is 170 mm from the crank-pin centre and the radius of gyration about an axis through the centre of mass is 148 mm. The engine runs at 320 rpm. Find the magnitude and the direction of the inertia force and the corresponding torque on the crankshaft when the angle turned by the crank is 140° from the inner dead centre.***

Solution

Analytical Method

$r = 220/2 = 110$ mm $\qquad N = 320$ rpm

$d = 125$ mm $\qquad l = 110 \times 4.5 = 495$ mm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} = 33.5 \text{ rad/s}$$

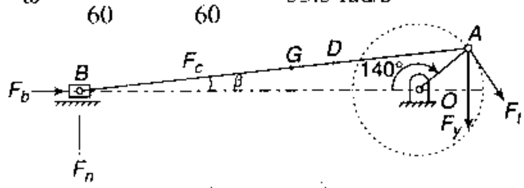


Fig. 13.15

Divide the mass of the connecting rod into two parts (refer Fig. 13.15).

Mass at crank pin, $m_a = 50 \times \left(\dfrac{495 - 170}{495}\right)$

$= 32.83$ kg

Mass at gudgeon pin, $m_b = 50 - 32.83 = 17.17$ kg

Total mass of reciprocating parts, $m = 30 + 17.17 = 47.17$ kg

Acceleration of the reciprocating parts,

$$f = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

As θ is more than 90°, it is negative or towards left and thus, the inertia force is towards right.

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

$$= 47.17 \times 0.11 \times (33.5)^2\left(\cos 140° + \frac{\cos 280°}{4.5}\right)$$

$= -4236$ N

Inertia torque due to reciprocating parts

$$T_b = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \text{[Eq. (13.21)]}$$

$$= -4236 \times 0.11\left(\sin 140^\circ + \frac{\sin 280^\circ}{2\sqrt{(4.5)^2 - \sin^2 140^\circ}}\right)$$

$= -248$ N.m

(clockwise as inertia force is towards left)

Correction couple due to assumed second mass of connecting rod at A,

$$\Delta T = m\alpha_c b(l - L) \quad \text{[Eq. (13.25)]}$$

where $b = 495 - 170 = 325$ mm

$l = 495$ mm

and $L = b + \frac{k^2}{b} = 325 + \frac{(148)^2}{325} = 392.4$ mm

$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

[Eq. (13.16)]

$$= -(33.5)^2 \sin 140^\circ\left[\frac{4.5^2 - 1}{(4.5 - \sin^2 140^\circ)^{3/2}}\right]$$

$= -157.17$ rad/s^2

$\therefore \Delta T = 50 \times (-157.17) \times 0.325 \times (0.495 - 0.3924)$

$= -262.04$ N.m

The direction of the correction couple will be the same as that of the angular acceleration, i.e., in the direction of decreasing angle β as discussed in Section 13.6. Thus, it is clockwise.

$\therefore$ correction torque on the crankshaft,

$$T_c = \Delta T \frac{\cos\vartheta}{\sqrt{n^2 - \sin^2\theta}}$$

$$= -262.04 \times \frac{\cos 140^\circ}{\sqrt{4.5^2 - \sin^2 140^\circ}}$$

$= 45.07$ N.m

The correction torque is to be deducted from the inertia torque on the crankshaft or as the force F_y due to ΔT (which is clockwise) is towards right of the crankshaft, the correction torque is clockwise.

Torque due to weight of mass at A,

$T_a = (m_a g)\, r \cos\theta$

$= 32.83 \times 9.81 \times 0.11 \times \cos 140^\circ$

$= -27.14$ N.m counter-clockwise

$\therefore$ total inertia torque on the crankshaft $= T_b - T_c + T_a$

$= -248 - 45.07 - 27.14$

$= 320.2$ clockwise

Graphical Method

Draw the configuration diagram OAB of the engine mechanism to a convenient scale (Fig. 13.16) and its velocity and acceleration diagrams by Klein's construction.

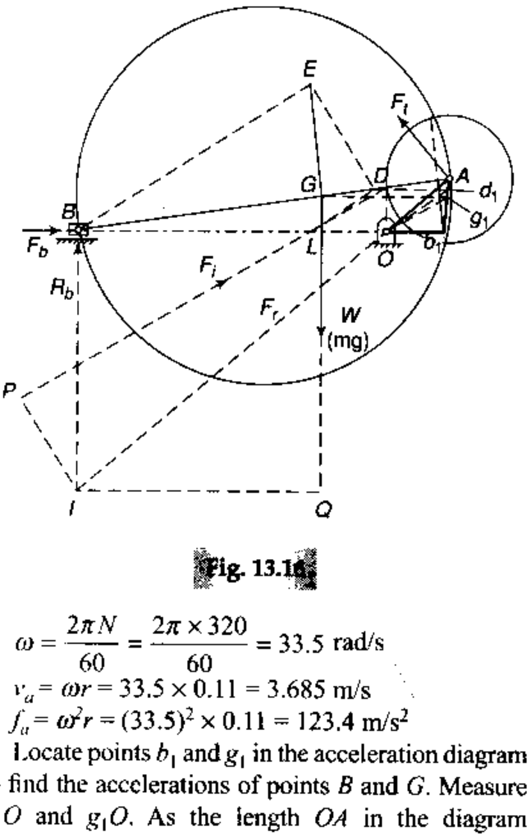


Fig. 13.16

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} = 33.5 \text{ rad/s}$$

$v_a = \omega r = 33.5 \times 0.11 = 3.685$ m/s

$f_a = \omega^2 r = (33.5)^2 \times 0.11 = 123.4$ m/s^2

Locate points b_1 and g_1 in the acceleration diagram to find the accelerations of points B and G. Measure b_1O and g_1O. As the length OA in the diagram represents the acceleration of A relative to O, i.e., 123.4 m/s^2, therefore, f_b can be obtained from

$$f_b = 123.4 \times \frac{b_1O}{OA}$$

It is found to be $f_b = 89.6$ m/s^2

Similarly, $f_g = 106.7$ m/s^2

$\therefore$ $F_b = m_b \times f_b = 30 \times 89.6 = 2688$ N

$F_i = m \times f_g = 50 \times 106.7 = 5335$ N

Complete the diagram of Fig. 13.16 as discussed in Section 13.10. Taking moments about I,

$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$

$F_t \times 0.64 = 2688 \times 0.340 + 5335 \times 0.138 + 50 \times 9.81 \times 0.322$

$F_t = 2825.1$ N

$T = F_t \times r = 2825.1 \times 0.11 = 310.7$ N.m

The difference of results by analytical and graphical methods can be due to practical error in drawing the Klein's construction and also because the equation used in analytical solution for acceleration are only approximate.

Example 13.11 *Figure 13.17(a) shows the link mechanism of a quick-return mechanism of the slotted lever type with the following dimensions:*

OA = 40 mm, OP = 20 mm, AR = 70 mm, RS = 30 mm.

The crank OA rotates at 210 rpm. The centres of mass of the links AR and RS are at their respective midpoints. The mass of the link AR is 15 kg and the radius of gyration is 265 mm about the centre of mass. The mass of the link RS is 6 kg and the radius of gyration is 90 mm about the centre of mass. The reciprocating mass is 5 kg at the slider S. Determine the torque required to be applied on the crank OP to overcome the inertia forces on the mechanism.

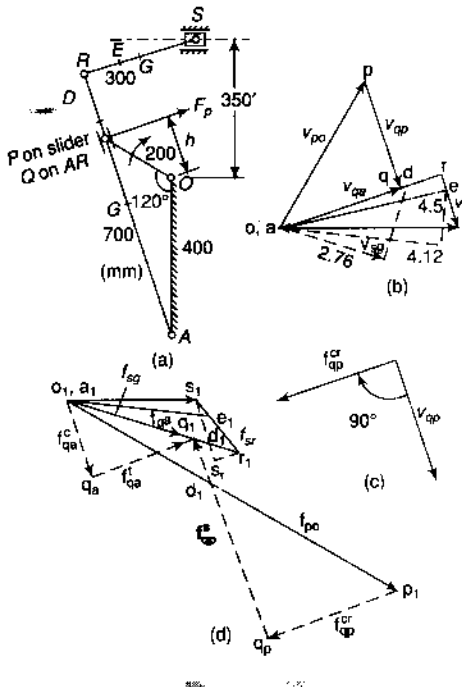


Fig. 13.17

Solution First of all, draw the configuration diagram to some suitable scale and find the dynamic equivalent masses on links AR and RS.

Link AR

$m = 15$ kg, $l = 700$ mm, $k = 265$ mm

Placing one dynamic mass at A and the other at D where D is located by

$$AD = AG_1 + \frac{k^2}{AG_1} = 350 + \frac{265^2}{350} = 550.6 \text{ mm}$$

Now, mass at D is calculated from,

$m_d \times AD = m \times GD$

or $m_d \times 550.6 = 15 \times 350$

or $m_d = 9.54$ kg and $m_a = 15 - 9.54 = 5.46$ kg

Link RS

$m = 6$ kg, $l = 300$ mm, $k = 90$ mm

Placing one dynamic mass at S and the other at E where E is located by

$$SE = SG' + \frac{k^2}{SG'} = 150 + \frac{90^2}{150} = 204 \text{ mm}$$

Now, mass at E is calculated from,

$m_e \times SE = m \times SG'$

or $m_e \times 204 = 6 \times 150$

or $m_e = 4.41$ kg and $m_s = 1.59$ kg

Total mass at S, $m_s = 1.59 + 5 = 6.59$ kg

Velocity and Acceleration Diagrams

Draw the velocity and acceleration diagrams as shown in Fig. 13.17. The procedure has been described in Example 3.7. Locate points **d** and **e** in the velocity diagram corresponding to points D and E respectively in the configuration diagram and in a similar way $\mathbf{d}_1$ and $\mathbf{e}_1$ in the acceleration diagram.

Acceleration of D = $\mathbf{a}_1\mathbf{d}_1 = 36.1$ m/s^2

Inertia force of mass at $D = 9.54 \times 36.1 = 344.8$ N

Velocity of D = **ad**

Taking its components along and $\perp$ to the inertia force at D,

Component along the force = 2.76 m/s

Work done = $344.8 \times 2.76 = 952$ N.m

Acceleration of $E = 36.63$ m/s^2

Inertia force of mass at $E = 4.41 \times 36.63 = 161.2$ N

Velocity of E = **oe**

Taking its components along and $\perp$ to inertia force at E,

Component along the force = 4.12 m/s

Work done = 161.2 × 4.12 = 664 N.m

Acceleration of S = 32.8 m/s^2

Inertia force of mass at S = 6.59 × 32.8 = 216 N

Velocity of S = **os** = 4.5 m/s

Work done = 216 × 4.5 = 973 N.m

Total work done = 952 + 664 + 973 = 2589 N.m (i)

This work must be equal to the torque to be applied to the crankshaft.

Let F_p be the force applied by the slider on the link AR which is ⊥ to AR.

Velocity of E = **op**

Its components along the force = **oq** = 3.26 m/s

Work done by $F_p = (F_p \times 3.26)$ (ii)

Equating (i) and (ii),

$F_p \times 3.26 = 2589$

∴ $F_p = 794$ N

Thus, the required torque = $F_p \times h$

h = 149.8 mm (on measurement from the configuration diagram)

$T = 568 \times 0.1498 = 119$ N.m

13.11 TURNING-MOMENT DIAGRAMS

During one revolution of the crankshaft of a steam engine or IC engine, the torque on it varies and is given by

$$T = F_t \times r$$

$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \text{(refer Eq. 3.21)}$$

where F is the net piston effort.

A plot of T vs. θ is known as the *turning-moment diagram*. The inertia effect of the connecting rod is, usually ignored while drawing these diagrams, but can be taken into account if desired.

As $T = F_t \times r$, a plot of F_t vs. θ (known as *crank effort diagram*) is identical to a turning-moment diagram.

The turning-moment diagrams for different types of engines are being given below:

1. Single-cylinder Double-acting Steam Engine

Figure 13.18 shows a turning-moment diagram for a single-cylinder double-acting steam engine. The crank angle θ is represented along the x-axis and the turning-moment along the y-axis. It can be observed that during the outstroke (ogp) the turning moment is maximum when the crank angle is a little less than 90° and zero when the crank angle is zero and 180°. A somewhat similar turning-moment diagram is obtained during the instroke (pkg).

Note that the area of the turning-moment diagram is proportional to the work done per revolution as the work is the product of the turning-moment and the angle turned.

The mean torque against which the engine works is given by

$$oe = \frac{\text{Area } ogpkp}{2\pi}$$

where oe is the mean torque and is the mean height of the turning-moment diagram.

When the crank turns from the angle oa to ob (Fig. 13.18), the work done by the engine is represented by the area $afghb$. But the work done against the resisting torque is represented by the area $afhb$. Thus, the engine has done more work than what has been taken from it.

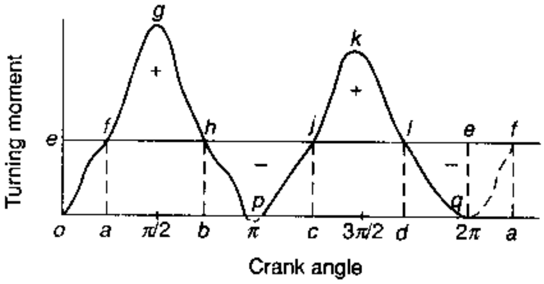


Fig. 13.18

The excess work is represented by the area *fgh*. This excess work increases the speed of the engine and is stored in the flywheel.

During the crank travel from *ob* or *oc*, the work needed for the external resistance is proportional to *bhjc* whereas the work produced by the engine is represented by the area under *hpj*. Thus, during this period, more work has been taken from the engine that is produced. The loss is made up by the flywheel which gives up some of its energy and the speed decreases during this period.

Similarly, during the period of crank travel from *oc* to *od*, excess work is again developed and is stored in the flywheel and the speed of the engine increases. During the crank travel from *od* to *oa*, the loss of work is made up by the flywheel and the speed again decreases.

The areas *fgh*, *hpj*, *jkl* and *lqf* represent fluctuations of energy of the flywheel. When the crank is at *b*, the flywheel has absorbed energy while the crank has moved from *a* to *b* and thereby, the speed of the engine is maximum. At *c*, the flywheel has given out energy while the crank has moved from *b* to *c* and thus the engine has a minimum speed. Similarly, the engine speed is again maximum at *d* and minimum at *a*. Thus, there are two maximum and two minimum speeds for the turning-moment diagram.

The greatest speed is the greater of the two maximum speeds and the least speed is the lesser of the two minimum speeds.

The difference between the greatest and the least speeds of the engine over one revolution is known as the *fluctuation of speed*.

2. Single-Cylinder Four-stroke Engine

In case of a four-stroke internal combustion engine, the diagram repeats itself after every two revolutions instead of one revolution as for a steam engine. It can be seen from the diagram (Fig. 13.19) that for the majority of the suction stroke, the turning moment is negative but becomes positive after the point *p*. During the compression stroke, it is totally negative. It is positive throughout the expansion stroke and again negative for most of the exhaust stroke.

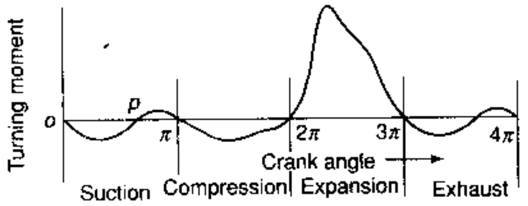


Fig. 13.19

3. Multi-Cylinder Engines

As observed in the foregoing paragraphs, the turning-moment diagram for a single-cylinder engine varies considerably and a greater variation of the same is observed in case of a four-stroke, single-cylinder engine. For engines with more than one cylinder, the total crankshaft torque at any instant is given by the sum of the torques developed by each cylinder at the instant.

For example, if an engine has two cylinders with cranks at 90°, the resultant turning moment diagram has a less variation than that for a single cylinder. In a three-cylinder engine having its cranks at 120°, the variation is still less.

Figure 13.20 shows the turning-moment diagram for a multicylinder engine. The mean torque line *ab* intersects the turning moment curve at *c*, *d*, *e*, *f*, *g* and *h*. The area under the wavy curve is equal to the area *oabk*. As discussed earlier, the

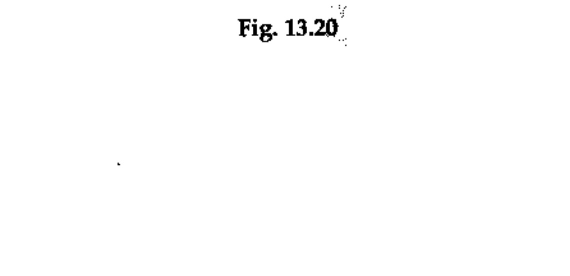


Fig. 13.20

speed of the engine will be maximum when the crank positions correspond to *d*, *f* and *h*, and minimum corresponding to *c*, *e* and *g*.

13.12 FLUCTUATION OF ENERGY

Let a_1, a_3, and a_5 be the areas in work units of the portions above the mean torque *ab* of the turning-moment diagram (Fig. 13.20). These areas represent quantities of energies added to the flywheel. Similarly, areas a_2, a_4 and a_6 below *ab* represent quantities of energies taken from the flywheel.

The energies of the flywheel corresponding to positions of the crank are as follows:

Crank position	*Flywheel energy*
c	E
d	$E + a_1$
e	$E + a_1 - a_2$
f	$E + a_1 - a_2 + a_3$
g	$E + a_1 - a_2 + a_3 - a_4$
h	$E + a_1 - a_2 + a_3 - a_4 + a_5$
c	$E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

From the two values of the energies of the flywheel corresponding to the position *c*, it is concluded that

$$a_1 - a_2 + a_3 - a_4 + a_5 - a_6 = 0$$

The greatest of these energies is the maximum kinetic energy of the flywheel and for the corresponding crank position, the speed is *maximum*.

The least of these energies is the least kinetic energy of the flywheel and for the corresponding crank position, the speed is *minimum*.

The difference between the maximum and minimum kinetic energies of the flywheel is known as the *maximum fluctuation of energy* whereas the ratio of this maximum fluctuation of energy to the work done per cycle is defined as the *coefficient of fluctuation of energy*.

The difference between the greatest speed and the least speed is known as the *maximum fluctuation of speed* and the ratio of the maximum fluctuation of speed to the mean speed is the *coefficient of fluctuation of speed*.

13.13 FLYWHEELS

A flywheel is used to control the variations in speed during each cycle of an engine. A flywheel of suitable dimensions attached to the crankshaft, makes the moment of inertia of the rotating parts quite large and thus, acts as a reservoir of energy. During the periods when the supply of energy is more than required, it stores energy and during the periods the requirements is more than the supply, it releases energy.

Let I = moment of inertia of the flywheel

ω_1 = maximum speed

ω_2 = minimum speed

ω = mean speed

E = kinetic energy of the flywheel at mean speed

e = maximum fluctuation of energy

K = coefficient of fluctuation of speed $= \dfrac{\omega_1 - \omega_2}{\omega}$

$$\text{Maximum fluctuation of energy, } e = \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$$

$$= \frac{1}{2} I(\omega_1^2 - \omega_2^2)$$

$$= I\left(\frac{\omega_1 + \omega_2}{2}\right)(\omega_1 - \omega_2)$$

$$= I\omega(\omega_1 - \omega_2)$$

$$= I\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

$$= I\omega^2 K$$

or

$$K = \frac{e}{I\omega^2} = \frac{e}{2 \times \frac{1}{2} I\omega^2} = \frac{e}{2E} \qquad (13.30)$$

Example 13.12 ***A flywheel with a mass of 3 kN has a radius of gyration of 1.6 m. Find the energy stored in the flywheel when its speed increases from 315 rpm to 340 rpm.***

Solution

$$\omega_1 = \frac{2\pi \times 340}{60} = 35.6 \text{ rad/s}$$

and $$\omega_2 = \frac{2\pi \times 315}{60} = 33 \text{ rad/s}$$

Additional energy stored

$$= \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} mk^2(\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} \times 3000 \times 1.6^2 \times (35.6^2 - 33^2)$$

= 684 900 N.m or 684.9 kN.m

or 684.9 kJ

Example 13.13 ***A flywheel absorbs 24 kJ of energy on increasing its speed of 210 rpm to 214 rpm. Determine its kinetic energy at 250 rpm.***

Solution Additional energy stored,

$$24\,000 = \frac{1}{2} I(\omega_1^2 - \omega_2^2)$$

$$\text{or } 24\,000 = \frac{1}{2} mk^2 \left(\frac{2\pi}{60}\right)^2 (214^2 - 210^2) \qquad \text{(i)}$$

Kinetic energy at 250 rpm,

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2 . \left(\frac{2\pi}{60}\right)^2 \times 250^2 \qquad \text{(ii)}$$

Dividing (ii) by (i), $\dfrac{E}{24\,000} = \dfrac{250^2}{214^2 - 210^2}$

or E = 884 430 N.m or 884.43 kN.m or 884.43 kJ

Example 13.14 ***A double-acting steam engine develops 56 kW of power at 210 rpm. The maximum and minimum speeds do not vary more than 1% of the mean speed and the excess energy is 30% of the indicated work per stroke. Determine the mass of the flywheel if the radius of gyration of the flywheel is 500 mm.***

Solution Work done per second = 56 000 W = 56 000 N.m

For a double-acting engine, the number of working strokes per minute = 2 × 210 = 420

Work done /stroke

$$= \frac{\text{Work done per second}}{\text{Number of working strokes/second}}$$

$$= \frac{56\,000}{420/60} = 8000 \text{ N.m}$$

Fluctuation of energy = 8000 × 0.3 = 2400 N.m

$$K = \frac{\omega_1 - \omega_2}{\omega} = \frac{1.01\omega - 0.99\omega}{\omega} = 0.02$$

Also, $K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$

or $0.02 = \frac{2400}{m \times 0.5^2 \times 22^2}$

or $m = 992$ kg

Example 13.15 *A flywheel fitted to a steam engine has a mass of 800 kg. Its radius of gyration is 360 mm. The starting torque of the engine is 580 N.m and may be assumed constant. Find the kinetic energy of the flywheel after 12 seconds.*

Solution Angular acceleration,

$$\alpha = \frac{T}{I} = \frac{T}{mk^2} = \frac{580}{800 \times 0.36^2} = 5.59 \text{ rad/s}^2$$

$$\omega_2 = \omega_1 + \alpha t = 0 + 5.59 \times 12 = 67.08 \text{ rad/s}$$

Kinetic energy

$$= \frac{1}{2}I\omega^2 = \frac{1}{2}mk^2\omega^2 = \frac{1}{2} \times 800 \times 0.36^2 \times 67.08^2$$

= 233 270 N.m or 233.27 kJ

Example 13.16 *The turning-moment diagram for a petrol engine is drawn to a vertical scale of 1 mm = 500 N.m and a horizontal scale of 1mm = 3°. The turning-moment diagram repeats itself after every half revolution of the crankshaft. The areas above and below the mean torque line are 260, –580, 80, –380, 870, and –250 mm². The rotating parts have a mass of 55 kg and radius of gyration of 2.1 m. If the engine speed is 1600 rpm, determine the coefficient of fluctuation of speed.*

Solution Let flywheel KE at $a = E$

(refer Fig. 13.21)

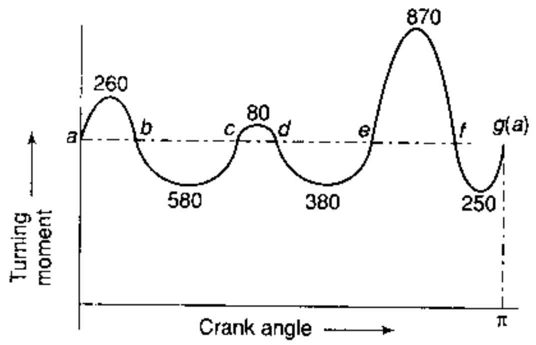


Fig. 13.21

at $b = E + 260$
at $c = E + 260 - 580 = E - 320$
at $d = E - 320 + 80 = E - 240$
at $e = E - 240 - 380 = E - 620$
at $f = E - 620 + 870 = E + 250$
at $g = E + 250 - 250 = E$

Maximum energy = $E + 260$ (at b)
Minimum energy = $E - 620$ (at e)
Maximum fluctuation of energy,

$e_{max} = (E + 260) - (E - 620) \times$ Hor. scale × Vert. scale

$$= 880 \times \left(3 \times \frac{\pi}{180}\right) \times 500$$

= 23 038 N.m

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2} = \frac{23\,038}{55 \times 2.1^2 \times \left(\frac{2\pi \times 1600}{60}\right)^2}$$

$K = 0.0034$ or 0.34%

Example 13.17 *A three-cylinder single-acting engine has its cranks at 120°. The turning-moment diagram for each cycle is a triangle for the power stroke with a maximum torque of 60 N.m at 60° after the dead centre of the corresponding crank. There is no torque on the return stroke. The engine runs at 400 rpm. Determine the*

(i) power developed

(ii) coefficient of fluctuation of speed if the mass of the flywheel is 10 kg and radius of gyration is 88 mm

(iii) coefficient of fluctuation of energy

(iv) maximum angular acceleration of flywheel

Solution The turning-moment diagram for each cylinder is shown in Fig. 13.22(a) and the resultant-turning moment diagram for the three combined cylinders is shown in Fig. 13.22(b).

(i) Work done/cycle = Area of three triangles

$$= 3 \times (60 \times \pi/2) = 90\pi$$

$$\text{Mean torque} = \frac{\text{Work done /cycle}}{\text{Angle turned}} = \frac{90\pi}{2\pi} = 45 \text{ N.m}$$

$$P = T\omega = 45 \times \frac{2\pi \times 400}{60} = 1885 \text{ W}$$

or 1.885 kW

(ii) As the area above or below the mean torque line is the maximum fluctuation of energy,

$$\therefore \quad e_{max} = \frac{60 \times \pi}{180} \times (60 - 45) \times \frac{1}{2}$$

$$= 2.5\pi \text{ N.m}$$

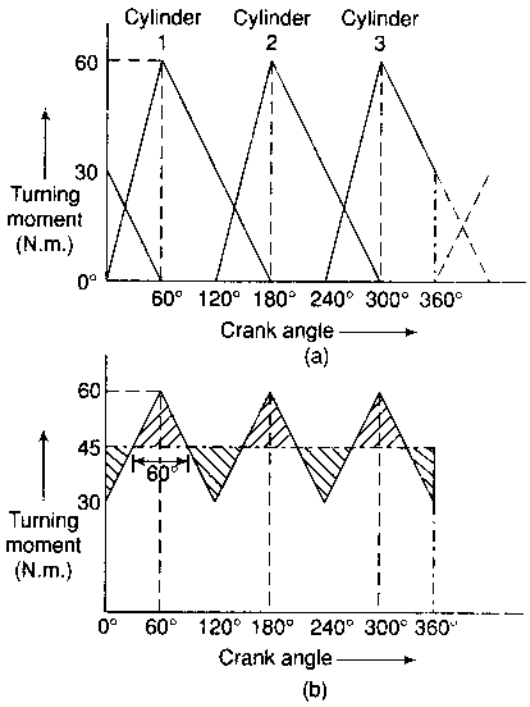


Fig. 13.22

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

$$= \frac{2.5\pi}{10 \times 0.088^2 \left(\frac{2\pi \times 400}{60}\right)^2}$$

$$= 0.0578 \quad \text{or } 5.78\%$$

(iii) Coefficient of fluctuation of energy,

$$K_e = \frac{\text{Maximum fluctuation of energy}}{\text{work done/cycle}}$$

$$= \frac{2.5\pi}{90\pi}$$

$$= 0.0278$$

(iv) Maximum fluctuation of torque

$$= 60 - 45 = 15 \text{ N.m}$$

$\therefore \quad \Delta T = 15$ N.m

or $\quad I\alpha = mk^2 \alpha = 15$

or $\quad 10 \times (0.088)^2 \times \alpha = 15$

or $\quad \alpha = 193.7 \text{ rad/s}^2$

Example 13.18 *In a single-acting four-stroke engine, the work done by the gases during the expansion stroke is three times the work done during the compression stroke. The work done during the suction and exhaust strokes is negligible. The engine develops 14 kW at 280 rpm. The fluctuation of speed is limited to 1.5% of the mean speed on either side. The turning-moment diagram during the compression and the expansion strokes may be assumed to be triangular in shape. Determine the inertia of the flywheel.*

Solution

$P = 14$ kW, $N = 280$ rpm, $K = 1.5\%$,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

It is a four-stroke engine, Thus, a cycle is completed in 4π radians. Thus the number of working strokes per minute is half the rpm, i.e., 140. The turning-moment diagram is shown in Fig. 13.23.

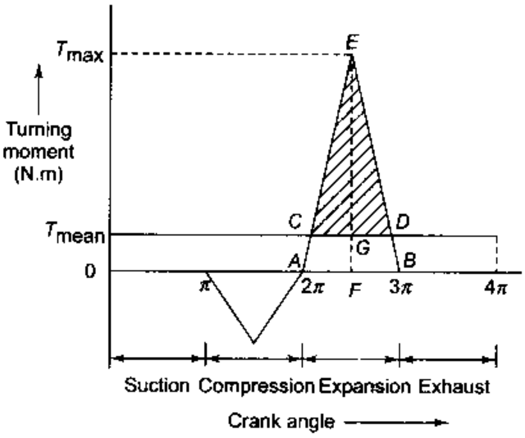


Fig. 13.23

Net energy produced/s = 14 000 N.m

Net energy produced/minute = 14 000 × 60 N.m

$$\text{Net energy produced/cycle} = \frac{14\,000 \times 60}{140} = 6\,000 \text{ N.m}$$

Now, during the compression stroke, the energy is absorbed whereas during the expansion stroke, it is produced.

Thus if E is the energy produced during the expansion stroke,

$$\text{Then } E - \frac{E}{3} = 6000 \quad \text{or} \quad E = 9000 \text{ N.m}$$

$$\text{Also } \frac{T_{max} \times \pi}{2} = 9000 \quad \text{or} \quad T_{max} = 5730 \text{ N.m}$$

and $T_{mean} \times 4\pi = 6000$; $\therefore$ $T_{mean} = 477.5$ N.m

In triangle ABE,

$$\frac{CD}{AB} = \frac{EG}{EF} = \frac{5730 - 477.5}{5730} = \frac{5252.5}{5730} = 0.9167$$

or $\quad CD = 0.9167 \times \pi = 2.88$ rad

and maximum fluctuation of energy,

$$e = \text{Area } CDE = \frac{CD \times EG}{2} = \frac{2.88 \times 5252.5}{2} = 7564 \text{ N.m}$$

$$K = \frac{e}{I\omega^2} \text{ or } 0.03 = \frac{7564}{I \times 29.32^2} \text{ or } I = 293.3 \text{ kg.m}^2$$

Example 13.19 ***The turning-moment diagram of a four-stroke engine is assumed to be represented by four triangles, the areas of which from the line of zero pressure are***

Suction stroke $= 440 \text{ mm}^2$

Compression stroke $= 1600 \text{ mm}^2$

Expansion stroke $= 7200 \text{ mm}^2$

Exhaust stroke $= 660 \text{ mm}^2$

Each mm² of area represents 3 N.m of energy. If the resisting torque is uniform, determine the mass of the rim of a flywheel to keep the speed between 218 and 222 rpm when the mean radius of the rim is to be 1.25 m.

Solution It is a four-stroke engine, Thus, a cycle is completed in 4π radians. The turning moment diagram is shown in Fig. 13.24.

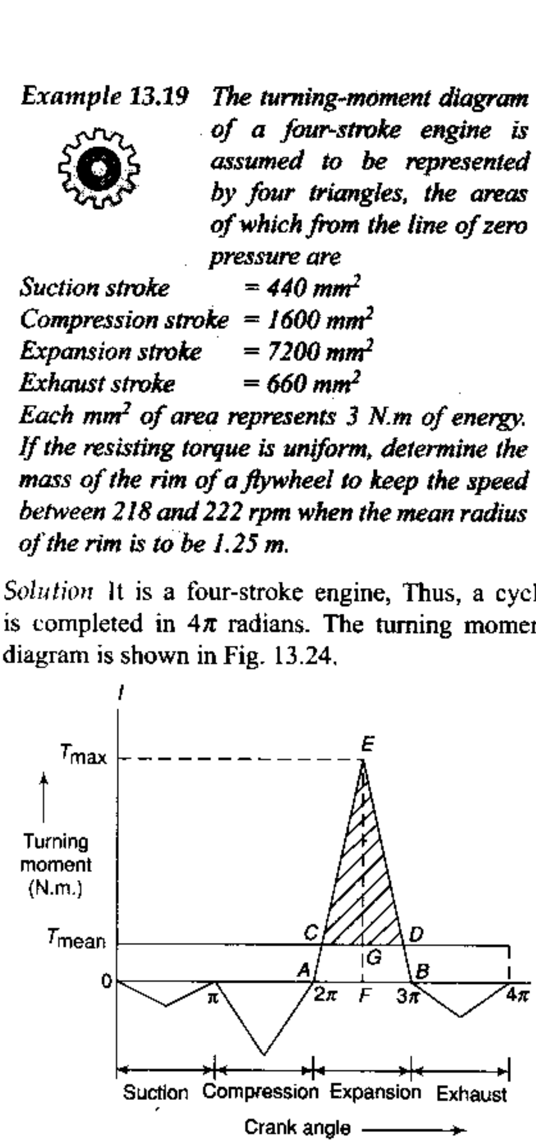


Fig. 13.24

The energy is produced only in the expansion stroke whereas in the other three strokes, it is spent only.

Net energy produced in one cycle

$= [7200 - (440 + 1600 + 660)] \times 3$

$= 13\,500$ N.m

Also $\quad T_{mean} \times 4\pi = 13\,500$

or $\quad T_{mean} = 1074$ N.m

Energy produced during expansion stroke = Area × Energy/mm² = 7200 × 3 = 21 600 N.m

As the area of the turning-moment diagram during the expansion stroke indicates the energy produced during the expansion stroke,

$$\therefore \quad \frac{T_{max} \times \pi}{2} = 21\ 600$$

or $\quad T_{max} = 13\ 751$ N.m

In triangle ABE,

$$\frac{CD}{AB} = \frac{EG}{EF} = \frac{13\ 751 - 1074}{13\ 751} = \frac{12\ 677}{13\ 751}$$

$$= 0.9219$$

or $\quad CD = 0.9219 \times \pi = 2.896$ rad

and $\quad$ maximum fluctuation of energy,

$$e = \text{Area } CDE = \frac{CD \times EG}{2} = \frac{2.896 \times 12\ 677}{2}$$

$$= 18\ 356 \text{ N.m}$$

$$\text{Now, } e = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$18\ 356 = \frac{1}{2} mk^2 (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} \times m \times 1.25^2 \left[\left(\frac{2\pi}{60}\right)^2 (222^2 - 218^2) \right]$$

$$= 15.0786 \text{ m}$$

$$m = \underline{1217.4 \text{ kg}}$$

Example 13.20 *The torque delivered by a two-stroke engine is represented by $T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ N.m where θ is the angle turned by the crank from the inner-dead centre. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine the*

(i) power developed

(ii) total percentage fluctuation of speed

(iii) angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner-dead centre

(iv) maximum angular acceleration and retardation of the flywheel

Solution For the expression for torque being a function of 2θ, the cycle is repeated every 180° of the crank rotation (Fig. 13.25).

(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta)\, d\theta$$

$$= \frac{1}{\pi}\left[1000\theta - \frac{300}{2}\cos 2\theta - \frac{500}{2}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{1}{\pi}[(1000\pi - 150 - 0) - (0 - 150 - 0)]$$

$$= 1000 \text{ N.m}$$

$$P = T\omega = 1000 \times \frac{2\pi \times 250}{60} = 26\ 180 \text{ W}$$

or $\underline{26.18 \text{ kW}}$

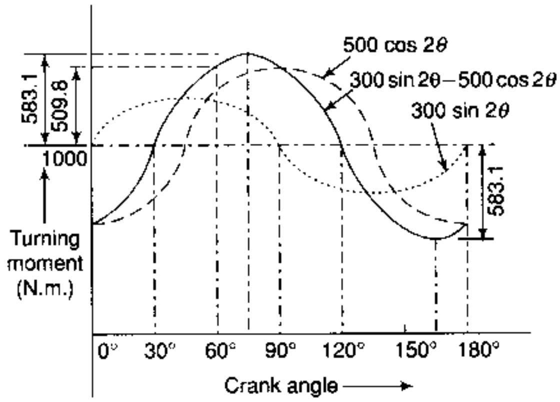


Fig. 13.25

(ii) At any instant, $\Delta T = T - T_{mean}$

$= (1000 + 300 \sin 2\theta - 500 \cos 2\theta) - 1000$

$= 300 \sin 2\theta - 500 \cos 2\theta$

ΔT is zero, when $300 \sin 2\theta - 500 \cos 2\theta = 0$

or $\quad 300 \sin 2\theta = 500 \cos 2\theta$

or $\quad \tan 2\theta = \frac{5}{3}$

or $\quad 2\theta = 59°$ or $239°$

$\theta = 29.5°$ or $119.5°$

$$e_{max} = \int_{29.5°}^{119.5°} \Delta T d\theta$$

$$= \int_{29.5°}^{119.5°} (300 \sin 2\theta - 500 \cos 2\theta)\, \theta$$

$= [-150\cos 2\theta - 250\sin 2\theta]_{29.5°}^{119.5°}$

$= 583.1$ N.m

$$K = \frac{e}{mk^2\omega^2}$$

$$= \frac{583.1}{400\times(0.4)^2\times\left(\frac{2\pi\times 250}{60}\right)^2}$$

$= 0.013\ 29$ or 1.329%

(iii) Acceleration or deceleration is produced by excess or deficit torque than the mean value at any instant.

$\Delta T = 300\sin 2\theta - 500\cos 2\theta$

when $\theta = 60°$,

$\Delta T = 259.8 - (-250) = 509.8$ N.m

or $I\alpha = mk^2\,\alpha = 509.8$

or $400\times(0.4)^2\times\alpha = 509.8$

or $\alpha = 7.966$ rad/s^2

(iv) For ΔT_{max} and ΔT_{min},

$$\frac{d}{d\theta}(\Delta T) = \frac{d}{d\theta}(300\sin 2\theta - 500\cos 2\theta) = 0$$

or $2\times 300\cos 2\theta + 2\times 500\sin 2\theta = 0$

or $600\cos 2\theta = -1000\sin 2\theta$

or $\tan 2\theta = -0.6$

or $2\theta = 149.04°$ and $329.04°$

or $\theta = 74.52°$ and $164.52°$

when $2\theta = 149.04°$, $T = 1583.1$ N.m,

$\Delta T = 583.1$ N.m

when $2\theta = 329.04°$, $T = 416.9$ N.m,

$\Delta T = -583.1$ N.m

As values of ΔT at maximum and minimum torque T are same, maximum acceleration is equal to maximum retardation.

or $\Delta T = mk^2\alpha = 583.1$

or $400\times(0.4)^2\times\alpha = 583.1$

Maximum acceleration or retardation, $\alpha = 9.11$ rad/s^2

Example 13.21 *A machine is coupled to a two-stroke engine which produces a torque of (800 + 180 sin 3θ) N.m, where θ is the crank angle. The mean engine speed is 400 rpm. The flywheel and the other rotating parts attached to the engine have a mass of 350 kg at a radius of gyration of 220 mm. Calculate the*

(i) power of the engine

(ii) total fluctuation of speed of the flywheel when the

(a) resisting torque is constant

(b) resisting torque is (800 + 80 sin θ) N.m

Solution

$m = 350$ kg $\qquad N = 400$ rpm

$k = 220$ mm $\qquad \omega = \frac{2\pi\times 400}{60} = 41.89$ rad/s

For the expression for torque being a function of 3θ, the cycle is repeated after every 120° of the crank rotation (Fig. 13.26).

$$\text{(i)}\quad T_{mean} = \frac{1}{2\pi/3}\int_0^{2\pi/3} T\,d\theta$$

$$= \frac{3}{2\pi}\int_0^{2\pi/3}(800 + 180\sin 3\theta)\,d\theta$$

$$= \frac{3}{2\pi}\left[800\theta - \frac{180}{3}\cos 3\theta\right]_0^{2\pi/3}$$

$= 800$ N.m

$P = T\omega = 800\times 41.89 = 33\ 512$ W

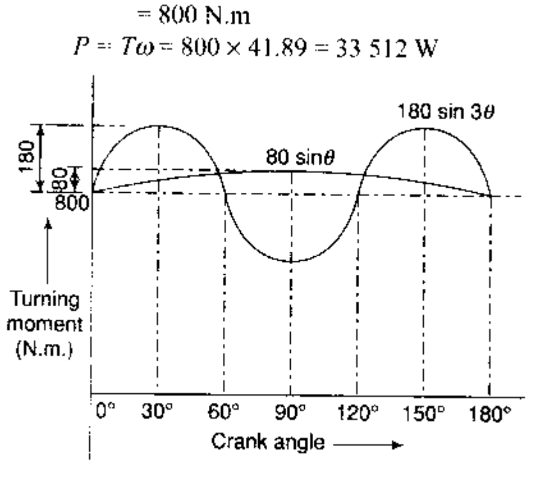


Fig. 13.26

(ii) (a) At any instant, $\Delta T = T - T_{mean}$

$= 800 + 180\sin 3\theta - 800$

$= 180\sin 3\theta$

ΔT is zero when $180 \sin 3\theta = 0$

or when $\sin 3\theta = 0$

or $3\theta = 0°$ or $180°$

or $\theta = 0°$ or $60°$

$$e_{max} = \int_{0°}^{60°} \Delta T dt$$

$$= \int_{0°}^{60°} (180 \sin 3\theta)\, d\theta$$

$$= \left[\frac{180 \cos 3\theta}{3}\right]_{0°}^{60°}$$

$= 120$ N.m

$$K = \frac{e}{mk^2\omega^2} = \frac{120}{350 \times (0.22)^2 \times (41.89)^2}$$

$= 0.00404$ or <u>0.404%</u>

(b) $\Delta T = T$ of engine $- T$ of machine

$= (800 + 180 \sin 3\theta) - (800 + 80 \sin\theta)$

$= 180 \sin 3\theta - 80 \sin \theta$

ΔT is zero when $180 \sin 3\theta - 80 \sin \theta = 0$

or $180 \sin 3\theta = 80 \sin \theta$

or $180 (3 \sin \theta - 4 \sin^3 \theta) = 80 \sin \theta$

or $3 - 4 \sin^2 \theta = \frac{80}{100} = 0.4444$

or $\sin^2\theta = 0.639$

or $\sin \theta = \pm 0.799$

or $\theta = \pm 53°$ and $\pm 127°$

$$e_{max} = \int_{53°}^{127°} \Delta T d\theta = \int_{53°}^{127°} (180 \sin 3\theta - 80 \sin\theta)\, d\theta$$

$$= \left[-\frac{180 \cos 3\theta}{3} + 80 \cos\theta\right]_{53°}^{127°}$$

$= -60 \cos 381° + 80 \cos 127° + 60 \cos 159° - 80 \cos 53°$

$= -208.3$ N.m

$$K = \frac{e}{mk^2\omega^2} = \frac{208.3}{350 \times (0.22)^2 \times (41.89)^2} = 0.007$$

$=$ <u>0.7%</u>

Example 13.22 *The torque delivered by a two-stroke engine is represented by $T = (1200 + 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta)$ N.m where θ is the angle turned by the crank from the inner-dead centre. The engine speed is 210 rpm. Determine the power of the engine and the minimum mass of the flywheel if its radius of gyration is 800 mm and the maximum fluctuation of speed is to be ± 1.5% of the mean.*

Solution

$k = 800$ mm $\qquad N = 210$ rpm

$K = 0.015 + 0.015 = 0.03$

The expression for torque being a function of θ, 2θ and 3θ the cycle is repeated after every 360° of the crank rotation (Fig. 13.27).

(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (1200 + 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left[1200\theta + 1400 \cos\theta + \frac{210}{2}\cos 2\theta + \frac{21}{2}\cos 3\theta\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}[(2400\pi + 1400 + 105 + 10.5) - (0 + 1400 + 105 + 10.5)]$$

$= 1200$ N.m

$$P = T\omega = 1200 \times \frac{2\pi \times 210}{60} = 26\,390 \text{ W}$$

or <u>26.39 kW</u>

(ii) At any instant, $\Delta T = T - T_{mean}$

$= (1200 + 1400 \sin\theta - 210 \sin 2\theta + 21 \sin 3\theta) - 1200$

$= 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta$

ΔT is zero when

$1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta = 0$

This will be so when θ is 180° or 360°. This can be easily seen from the plot of the turning moment diagram.

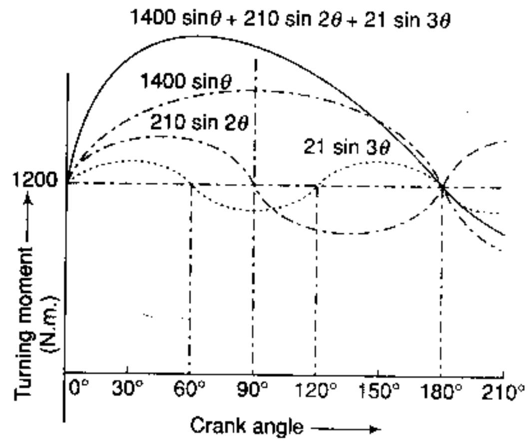


Fig. 13.27

$$e_{max} = \int_0^{\pi} \Delta T dt = \int_0^{\pi} (1200 + 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta)\, d\theta$$

$$= \left[1400 \cos\theta + \frac{210}{2}\cos 2\theta + \frac{21}{2}\cos 3\theta\right]_0^{\pi}$$

$$= [(-1400 + 105 - 10.5) - (1400 + 105 + 10.5)]$$

$$= 2821 \text{ N.m}$$

Now, $K = \dfrac{e}{mk^2\omega^2}$

$$0.03 = \frac{2821}{m \times (0.8)^2 \times \left(\dfrac{2\pi \times 210}{60}\right)^2}$$

$$0.03 = \frac{2821}{\mu \times 3.095}$$

$$m = \underline{303.8 \text{ kg}}$$

Example 13.23 *In a machine, the intermittent operations demand the torque to be applied as follows:*

- *During the first half-revolution, the torque increases uniformly from 800 N.m to 3000 N.m*
- *During the next one revolution, the torque remains constant*
- *During the next one revolution, the torque decreases uniformly from 3000 N.m to 800 N.m*
- *During last half-revolution, the torque remains constant.*

Thus, a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 250 rpm. A flywheel of mass 1800 kg and radius of gyration of 500 mm is fitted to the shaft. Determine the

(i) power of the motor

(ii) total fluctuation of speed of the machine shaft

Solution

$m = 1800$ kg $\quad N = 250$ rpm

$k = 500$ mm

(a) Refer Fig. 13.28.

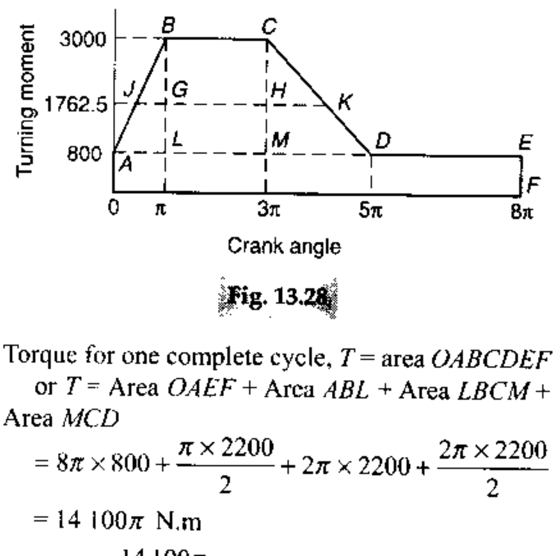


Fig. 13.28

Torque for one complete cycle, T = area $OABCDEF$

or T = Area $OAEF$ + Area ABL + Area $LBCM$ + Area MCD

$$= 8\pi \times 800 + \frac{\pi \times 2200}{2} + 2\pi \times 2200 + \frac{2\pi \times 2200}{2}$$

$$= 14\,100\pi \text{ N.m}$$

$$T_{mean} = \frac{14\,100\pi}{8\pi} = 1762.5 \text{ N.m}$$

$$P = T_m\omega = 1762.5 \times \frac{2\pi \times 250}{60} = 46\,142 \text{ W}$$

or $\underline{46.143 \text{ kW}}$

(ii) $JG = AL \times \dfrac{BG}{BL} = \pi \times \dfrac{3000 - 1762.5}{3000 - 800}$

$$= 1.767$$

$$HK = MD \times \frac{CH}{CM} = 2\pi \times \frac{3000 - 1762.5}{3000 - 800}$$

$$= 3.534$$

The fluctuation of energy is equal to the area above the mean torque line.

e = Area $JBCK$ = area JBG + area $GBCH$ + area HCK

$$= (3000 - 1762.5)\left[\frac{1.767}{2} + 2\pi + \frac{3.534}{2}\right]$$

$$= 11\ 055\ \text{N.m}$$

$$K = \frac{e}{mk^2\omega^2}$$

$$= \frac{11\ 055}{1800 \times (0.5)^2 \times \left(\frac{2\pi \times 250}{60}\right)^2}$$

$$= 0.0358 \text{ or } \underline{3.58\%}$$

13.14 DIMENSIONS OF FLYWHEEL RIMS

The inertia of a flywheel is provided by the hub, spokes and the rim. However, as the inertia due to the hub and the spokes is very small, usually it is ignored. In case it is known, it can be taken into account.

Consider a rim of the flywheel as shown in Fig. 13.29.

Let ω = angular velocity

r = mean radius

t = thickness of the rim

ρ = density of the material of the rim

Consider an element of the rim,

Centrifugal force on the element/unit length = $[\rho(r.d\theta)t].r\omega^2$

Total vertical force/unit length

$$= \int_0^\pi \rho.r^2.d\theta.t.\omega^2 \sin\theta = \rho.r^2.t.\omega^2 \int_0^\pi \sin\theta.d\theta$$

$$= \rho.r^2.t.\omega^2(-\cos\theta)\,_0^\pi = 2\rho.r^2.t.\omega^2$$

Let σ = circumferential stress induced in the rim (Circumferential stress is also known as *hoop stress*.)

Then for equilibrium, $\sigma(2t).1 = 2\rho.r^2.t.\omega^2$

$$\sigma = \rho.r^2\omega^2 = \rho.v^2 \qquad (13.31)$$

The above relation provides the limiting tangential velocity at the mean radius of the rim of the flywheel. Then the diameter can be calculated from the relation,

$$v = \pi dN/60.$$

Also, mass = density × volume = density × circumference × cross-sectional area

or $\quad m = \rho.\pi d.b.t \qquad (13.32)$

The relation can be used to find the width and the thickness of the rim.

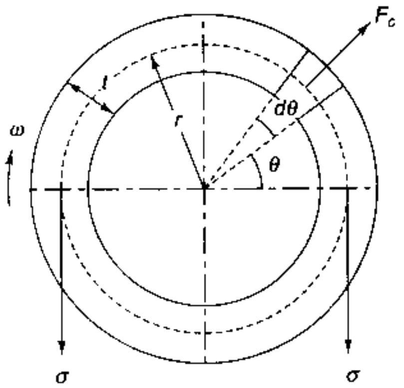
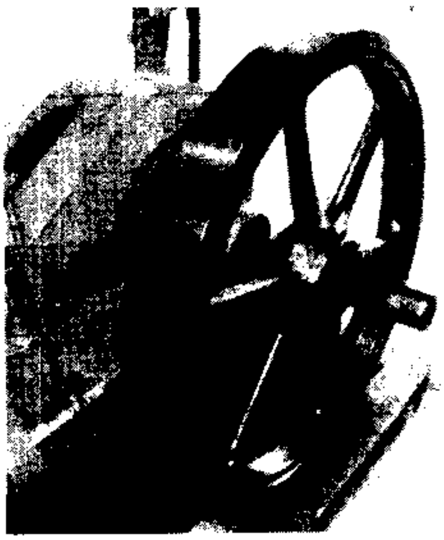


Fig. 13.29



Flywheel of a diesel engine

Example 13.24 ***The turning-moment diagram for a multicylinder engine has been drawn to a vertical scale of 1 mm = 650 N.m and a horizontal scale***

of 1 mm = 4.5°. The areas above and below the mean torque line are –28, + 380, –260, + 310, –300, +242, –380, +265 and –229 mm^2.

The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m^3 and width of the rim is 4.5 times its thickness. The centrifugal stress (hoop stress) in the rim material is limited to 6 N/mm^2. Neglecting the effect of the boss and arms, determine the diameter and cross section of the flywheel rim.

Solution

$\rho = 7000$ kg/m^3 $\quad\quad \sigma = 6 \times 10^6$ N/m^2

$N = 400$ rpm $\quad\quad K = 0.018 + 0.018 = 0.036$

$b = 4.5t$

Now,

$$\sigma = \rho v^2 \qquad \text{(Eq. 13.31)}$$

$$6 \times 10^6 = 7000 \times v^2$$

$$v = 29.28 \text{ m/s}$$

or $$\frac{\pi dn}{60} = \frac{\pi \times d \times 400}{60} = 29.28$$

or $$d = 1.398 \text{ m}$$

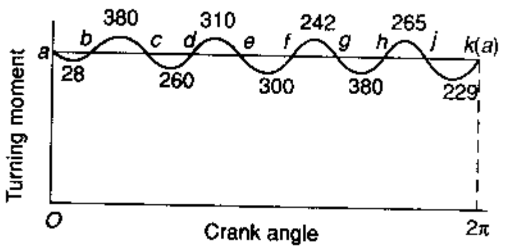


Fig. 13.30

Refer the turning-moment diagram of Fig. 13.30,
Let the flywheel KE at $a = E$

at $b = E - 28$

at $c = E - 28 + 380 = E + 352$

at $d = E + 352 - 260 = E + 92$

at $e = E + 92 + 310 = E + 402$

at $f = E + 402 - 300 = E + 102$

at $g = E + 102 + 242 = E + 344$

at $h = E + 344 - 380 = E - 36$

at $j = E - 36 + 265 = E + 229$

at $k = E + 229 - 229 = E$

Maximum energy = $E + 402$ (at e)

Minimum energy = $E - 36$ (at h)

Maximum fluctuation of energy,

$$e_{max} = (E + 402) - (E - 36) \times \text{hor. scale} \times \text{vert. scale}$$

$$= 438 \times \left(4.5 \times \frac{\pi}{180}\right) \times 650$$

$$= 22\,360 \text{ N.m}$$

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

$$0.036 = \frac{22\,360}{m\left(\frac{1.398}{2}\right)^2\left(\frac{2\pi \times 400}{60}\right)^2}$$

$$m = 724.5 \text{ kg}$$

or density × volume = 724.5

or $\rho \times (\pi d) \times t \times 4.5t = 724.5$

or $7000 \times \pi \times 1.398 \times t \times 4.5t = 724.5$

or $t = 0.0512$ m or 51.2 mm

$b = 4.5 \times 51.2 = 230.3$ mm

Example 13.25 ***The speed variation of an Otto cycle engine during the power stroke is limited to 0.8% of the mean speed on either side. The engine develops 40 kW of power at a speed of 130 rpm with 65 explosions per minute. The work done during the power stroke is 1.5 times the work done during the cycle. If the hoop stress in the rim of the flywheel is not to exceed 3.5 MPa and the width is three times the thickness, determine the mean diameter and the cross section of the rim. Assume that the energy stored by the flywheel is 1.1 times the energy stored by the rim and the density of the rim material is 7300 kg/m^3. The turning-moment diagram during the expansion stroke may be assumed to be triangular in shape.***

Solution As the number of explosions are half the speed of the engine, it is a four-stroke engine and the cycle is completed in 4π radians. The turning-moment diagram is shown in Fig. 13.31.

$$P = \frac{2\pi NT}{60} \text{ or } 40\,000 = \frac{2\pi \times 130 \times T_{mean}}{60}$$

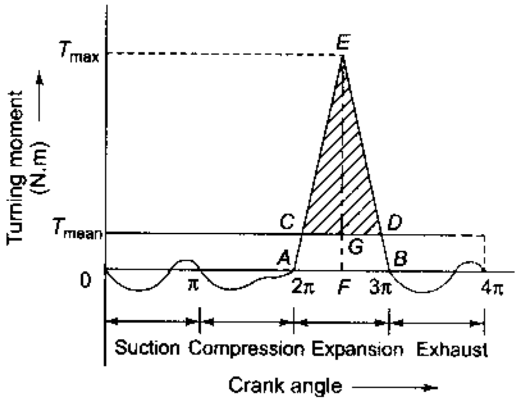


Fig. 13.31

or $\quad T_{mean} = 2938.2$ N.m

or $\quad$ Energy produced per cycle $= 2938.2 \times 4\pi$ $= 36\,923$ N.m

Energy produced during expansion stroke $= 36\,923 \times 1.5 = 55\,385$ N.m

The work done or the energy produced during the power stroke $= \frac{T_{max} \times \pi}{2} = 55\,385$

or $\quad T_{max} = 35\,259$ N.m

In triangle ABE

$$\frac{CD}{AB} = \frac{EG}{EF} = \frac{35\,259 - 2938.2}{35\,259}$$

$$= \frac{32\,320.8}{35\,259} = 0.9167$$

or $\quad CD = 0.9167 \times \pi = 2.88$ rad

and maximum fluctuation of energy,

$$e = \text{Area } CDE = \frac{CD \times EG}{2} = \frac{2.88 \times 32\,320.8}{2}$$

$= 46\,542$ N.m

From strength considerations, the hoop stress,

$\sigma = \rho v^2$ or $3.5 \times 10^6 = 7000 \times v^2$ or $v = 22.36$ m/s

or $\quad \frac{\pi dN}{60} = \frac{\pi \times d \times 130}{60} = 22.36$

or $\quad d = 3.285$ m

Energy stored in the rim $= 46\,542/1.1$ $= 42\,311$ N.m

Now, $K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$ or 0.016

$$= \frac{42\,311}{m\left(\frac{3.285}{2}\right)^2\left(\frac{2\pi \times 130}{60}\right)^2} \text{ or } m = 5289 \text{ kg}$$

or $\quad$ density $\times$ volume $= 5289$

or $\quad \rho \times (\pi d) \times t \times 4.5t = 5289$

or $\quad 7300 \times \pi \times 3.285 \times t \times 3t = 5289$

or $\quad t = 0.153$ m or 153 mm

and $\quad b = 3 \times 153 = 459$ mm

13.15 PUNCHING PRESSES

From the previous discussion, it can be observed that when the load on the crankshaft is constant or varies and the input torque varies continuously during a cycle, a flywheel is used to reduce the fluctuations of speed. A flywheel can perform the same purpose in a punching press or a riveting machine in which the torque available is constant but the load varies during the cycle. Figure 13.32 shows the sketch of a punching press. It is a slider-crank mechanism in which a punch replaces the slider. A motor provides a constant torque to the crankshaft through a flywheel. It may be observed that the actual punching process is performed only during the downward stroke of the punch and that also for a limiting period when the punch travels through the thickness of the plate. Thus, the load is applied during the actual punching process only and during the rest of the downward stroke and the return stroke, there is no load on the crankshaft. In the absence of a flywheel, the decrease in the speed of the crankshaft will be very large during the actual punching period whereas it will increase to a much higher value during the no-load period as the motor will continue to supply the energy all the time.

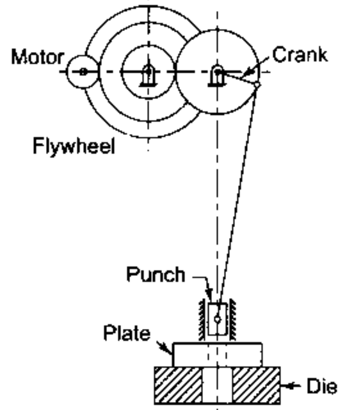


Fig. 13.32

Example 13.26 *A riveting machine is driven by a motor of 3 kW. The actual time to complete one riveting operation is 1.5 seconds and it absorbs 12 kN.m of energy.* ***The moving parts including the flywheel are equivalent to 220 kg at 0.5 m radius. Determine the speed of the flywheel immediately after riveting if it is 360 rpm before riveting. Also, find the number of rivets closed per minute.***

Solution

$P = 3$ kW, $m = 220$ kg, $k = 0.5$ m,

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}$$

Energy required/riveting = 12 000 N.m

Energy supplied by the motor in 1 seconds = 3000 N.m

∴ energy supplied by the motor in 1.5 seconds = 3000 × 1.5 = 4500 N.m

Energy supplied by the flywheel

e = energy required/hole – energy supplied by the motor in 1.5 s

= 12000 – 4500 = 7500 N.m

Also $e = \frac{1}{2} I (\omega_1^2 - \omega_2^2) = \frac{1}{2} mk^2 (\omega_1^2 - \omega_2^2)$

or $\quad 7500 = \frac{1}{2} \times 220 \times 0.5^2 (37.7^2 - \omega_2^2)$

or $\quad 37.7^2 - \omega_2^2 = 272.7$ or $\omega_2 = 33.89$ rad/s

or $\quad N_2 = \frac{33.89 \times 60}{2\pi}$ 323.6 rpm

Now, energy supplied by the motor in one minute = 3000 × 60 N.m

Energy required/riveting = 12 000 N.m

∴ number of rivets closed /minute

$$= \frac{3000 \times 60}{12\,000} = 15$$

Example 13.27 ***A punching machine carries out 6 holes per minute. Each hole of 40-mm diameter in 35-mm thick plate requires 8 N.m of energy/mm² of the sheared area. The punch has a stroke of 95 mm. Find the power of the motor required if the mean speed of the flywheel is 20 m/s. If total fluctuation of speed is not to exceed 3% of the mean speed, determine the mass of the flywheel.***

Solution

$d = 40$ mm $\quad K = 0.03$

$t = 35$ mm $\quad$ Stroke = 95 mm

$v = 20$ m/s

As 6 holes are punched in one minute, time required to punch one hole is 10 s.

Energy required/hole or energy supplied by the motor in 10 seconds

= area of hole × energy required /mm²

$= \pi dt \times 8$

= 35 186 N.m

∴ energy supplied by the motor in 1 seconds

$= \dfrac{35186}{10} = 3518.6$ N.m

Power of the motor, $P = 3518.6$ W or <u>3.5186 kW</u>

The punch travels a distance of 190 mm (upstroke + downstroke) in 10 seconds (6 holes are punched in 1 minute).

∴ Actual time required to punch a hole in 35-mm thick plate $= \dfrac{10}{190} \times 35 = 1.842$ s

Energy supplied by the motor in 1.842 s

$= 3518.6 \times 1.842 = 6481$ N.m

Energy supplied by the flywheel

e = energy required/hole – energy supplied by the motor in 1.842 s

$= 35\,186 - 6481 = 28\,705$ N.m

or $\quad 2KE = 28\,705$

∴ $\quad 2 \times 0.03 \times E = 28\,705$

or $\quad E = 478\,417$

or $\quad \dfrac{1}{2} mv^2 = 478\,417$

or $\quad \dfrac{1}{2} m(20)^2 = 478\,417$

or $\quad m = \underline{2392}$ kg

Example 13.28 *A punching machine punches 20 holes of 30-mm diameter in 20-mm thick plates per minute. The actual punching operation is done in 1/10th of a revolution of the crankshaft. Ultimate shear strength of the steel plates is 280 N/mm². The coefficient of fluctuation of speed is 0.12. The flywheel with a maximum diameter of 1.6 m rotates at 12 times the speed of the crankshaft. Determine the*

(i) power of the motor assuming the mechanical efficiency to be 92%

(ii) cross section of the flywheel rim if width is twice the thickness

The flywheel is of cast iron with a working tensile stress of 6 N/mm² and a density of 7000 kg/m³. The hub and the spokes of the flywheel may be assumed to deliver 8% of the rotational inertia of the wheel.

Solution $\quad d = 30$ mm, $t = 20$ mm, $\tau_u = 280$ N/mm², $n = 20$, $\eta = 0.92$, $K = 0.12$, $\rho = 7000$ kg/m³, $D = 1.6$ m, $k = D/2 = 0.8$ m

Maximum shear force required/punching

= area × ultimate shear stress

$= \pi \times 30 \times 20 \times 280 = 527\,800$ N

Energy required per punching or stroke

= Average shear force × displacement (thickness)

$= \dfrac{527\,800}{2} \times 0.02 = 5278$ N.m

Energy required per second = Energy per stroke × No. of strokes per second

$= 5278 \times \dfrac{20}{60} = 1759.3$

Power of the motor

= Energy required per second/Efficiency

$= \dfrac{1759.3}{0.92} = 1912$ W or 1.912 kW

As the actual punching is done in 1/10th of a cycle, the energy is stored in the flywheel during the 9/10th of the cycle.

∴ maximum fluctuation of energy = energy stored in the flywheel/stroke

$= 5278 \times 0.9 = 4750$ N.m

Since the hub and the spokes of the flywheel delivers 8% of the rotational inertia of the wheel, maximum fluctuation of energy provided by the rim $= 4750 \times 0.8 = 4370$ N.m

Mean angular speed of the flywheel

$= \dfrac{2\pi(20 \times 12)}{60} = 25.13$ rad/s

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

or $\quad 0.12 = \dfrac{4370}{m \times 0.8^2 \times 25.13^2}$

or $\quad m = 90$ kg

or $\quad$ Density × volume = 90

or $\quad \rho \times (\pi D) \times t \times 4.5t = 90$

or $\quad 7000 \times \pi \times 1.6 \times t \times 2t = 90$

or $\quad t = 0.0358$ m or 35.8 mm

or $\quad b = 2 \times 35.8 = 71.6$ mm

Summary

1. Dynamic forces are associated with accelerating masses. As all machines have some accelerating parts, dynamic forces are always present when the machines operate.
2. *D'Alembert's* principle states that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.
3. In graphical solutions, it is possible to replace inertia force and inertia couple by an *equivalent offset inertia force* which can account for both. This is done by displacing the line of action of the inertia force from the centre of mass.
4. The sense of angular acceleration of the connecting rod is such that it tends to reduce the angle of the connecting rod with the line of stroke.
5. *The piston effort* is the net or effective force applied on the piston.
6. Inertia force on the piston,
$$F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$
7. *Crank effort* is the net effort (force) applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft.
8. Turning moment due to force F on the piston
$$= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$
9. *A dynamically equivalent system* means that the rigid link is replaced by a link with two point masses in such a way that it has the same motion as the rigid link when subjected to the same force, i.e., the centre of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.
10. The distributed mass of a rod can be replaced by two point masses to have the same dynamical properties if the sum of the two masses is equal to the total mass, the combined centre of mass coincides with that of the rod and the moment of inertia of two point masses about the perpendicular axis through their combined centre of mass is equal to that of the rod.
11. In the analysis of the connecting rod, the two point masses are considered to be located at the centre of the two end bearings and then a correction is applied for the error involved.
12. A plot of T vs. θ is known as the *turning-moment diagram*.
13. The difference between the maximum and minimum kinetic energies of the flywheel is known as the *maximum fluctuation of energy*.
14. The difference between the greatest speed and the least speed is known as the *maximum fluctuation of speed*.
15. A flywheel is used to control the variations in speed during each cycle of an engine.
16. Coefficient of fluctuation of speed is given by
$$K = \frac{e}{I\omega^2} = \frac{e}{2E}$$

Exercises

1. State and explain D' Alembert's principle.
2. What do you mean by equivalent offset inertia force? Explain.
3. Derive an expression for the angular acceleration of the connecting rod of a reciprocating engine.
4. What is meant by piston effort and crank effort?
5. Derive a relation for the turning moment at the crankshaft in terms of piston effort and the angle turned by the crank.
6. What do you mean by dynamical equivalent system? Explain.
7. In what way is the inertia of the connecting rod of a reciprocating engine taken into account?
8. When and why is the correction couple applied while considering the inertia of the connecting rod of a reciprocating engine?
9. Describe the graphical method of considering the inertia of the connecting rod of a reciprocating engine.
10. What are turning-moment diagrams? Why are they drawn?
11. Define the terms coefficient of fluctuation of energy and coefficient of fluctuation of speed.
12. What is a flywheel? What is its use?
13. Find a relation for the coefficient of fluctuation of speed in terms of maximum fluctuation of energy

and the kinetic energy of the flywheel at mean speed.

14. In a four-link mechanism *ABCD*, the link *AB* revolves with an angular velocity of 10 rad/s and angular acceleration of 25 rad/s^2 at the instant when it makes an angle of 45° with *AD*, the fixed link. The lengths of the links are

    AB = *CD* = 800 mm, *BC* = 1000 mm, and *AD* = 1500 mm

    The mass of the links is 4 kg/m length. Determine the torque required to overcome the inertia forces, neglecting the gravitational effects. Assume all links to be of uniform cross-sections.

    (82.2 N.m)

15. The following data relate to a four-link mechanism:

Link	*Length*	*Mass*	*MOI about an axis through centre of mass*
AB	60 mm	0.2 kg	80 kg.mm²
BC	200 mm	0.4 kg	1600 kg.mm²
CD	100 mm	0.6 kg	400 kg.mm²
AD	140 mm		

    AD is the fixed link. The centres of mass for the links *BC* and *CD* lie at their midpoints whereas the centre of mass for link *AB* lies at *A*. Find the drive torque on the link *AB* at the instant when it rotates at an angular velocity of 47.5 rad/s counter-clockwise and ∠*DAB* 135°. Neglect gravity effects.

    (1.96 N.m clockwise)

16. The effective steam pressure on the piston of a vertical steam engine is 200 kN/m² when the crank is 40° from the inner-dead centre on the downstroke. The crank length is 300 mm and the connecting rod length is 1200 mm. The diameter of the cylinder is 800 mm. What will be the torque on the crankshaft if the engine speed is 300 rpm and the mass of the reciprocating parts 250 kg?

    (9916 N.m)

17. The length of the connecting rod of a gas engine is 500 mm and its centre of gravity lies at 165 mm from the crank-pin centre. The rod has a mass of 80 kg and a radius of gyration of 182 mm about an axis through the centre of mass. The stroke of piston is 225 mm and the crank speed is 300 rpm. Determine the inertia force on the crankshaft when the crank has turned (a) 30°, and (b) 135° from the inner-dead centre.

    (302.3 N.m; 226.7 N.m)

18. The connecting rod of an IC engine is 450 mm long and has a mass of 2 kg. The centre of mass of the rod is 300 mm from the small end and its radius of gyration about an axis through this centre is 175 mm. The mass of the piston and the gudgen pin is 2.5 kg and the stroke is 300 mm. The cylinder diameter is 115 mm. Determine the magnitude and the direction of the torque applied on the crankshaft when the crank is 40° and the piston is moving away from the inner dead centre under an effective gas pressure of 2 N.mm². The engine speed is 1000 rpm.

    (1994 N.m)

19. The connecting rod of a vertical high-speed engine is 600 mm long between centres and has a mass of 3 kg. Its centre of mass lies at 200 mm from the big end bearing. When suspended as a pendulum from the gudgeon pin axis, it makes 45 complete oscillations in 30 seconds. The piston stroke is 250 mm. The mass of the reciprocating parts is 1.2 kg. Determine the inertia torque on the crankshaft when the crank makes an angle of 140° with top-dead centre. The engine speed is 1500 rpm.

    (361.7 N.m)

20. The turning-moment diagram for a petrol engine is drawn to a vertical scale of 1 mm to 6 N.m and a horizontal scale of 1 mm to 1°. The turning moment repeats itself after every half revolution of engine. The areas above and below the mean torque line are

    305, 710, 50, 350, 980 and 275 mm²

    The rotating parts amount to a mass of 40 kg at a radius of gyration of 140 mm. Calculate the coefficient of fluctuation of speed if the speed of the engine is 1500 rpm.

    (0.55%)

21. Determine the energy released by a flywheel having a mass of 2 kN and radius of gyration of 1.2 m when its speed decreases from 460 rpm to 435 rpm.

    (353.59 kJ)

22. A flywheel is used to give up 18 kJ of energy in reducing its speed from 100 rpm to 98 rpm. Determine its kinetic energy at 140 rpm.

    (890.9 kJ)

23. The cranks of a three-cylinder single-acting engine are set equally at 120°. The engine speed is 540 rpm. The turning-moment diagram for each cylinder is a triangle for the power stroke with a maximum torque of 100 N.m at 60° after dead-centre of the corresponding crank. On the return stroke, the torque is sensibly zero. Determine the
    (a) power developed
    (b) coefficient of fluctuation of speed if the

flywheel has a mass of 7.5 kg with a radius of gyration of 65 mm
(c) coefficient of fluctuation of energy
(d) maximum angular acceleration of the flywheel

(4.24 kW; 12.9%; 2.78%; 789 rad/s^2)

24. A certain machine requires a torque of (1500 + 200 sin θ) N.m to drive it, where is the angle of rotation of the shaft. The machine is directly coupled to an engine which produces a torque of (1500 + 200 sin 2θ) N.m. The flywheel and the other rotating parts attached to the engine have a mass of 300 kg at a radius of gyration of 200 mm. If the mean speed is 200 rpm, find the
(a) fluctuation of energy
(b) total percentage fluctuation of speed
(c) maximum and the minimum angular acceleration of the flywheel and the corresponding shaft positions

(490 N.m; 9.3%; 10 rad/s^2, 35.5; 33.35 rad/s^2, 127.9°)

25. A constant torque motor of 2.5 kW drives a riveting machine. The mass of the moving parts including the flywheel is 125 kg at 700 mm radius of gyration. One riveting operation absorbs 1 kJ of energy and takes one second. Speed of the flywheel is 240 rpm before riveting. Determine the
(ii) number of rivets closed per hour, and
(iii) reduction in speed after the riveting operation.

(900; 52.7 rpm)

26. A machine tool performs an operation intermittently. It is driven continuously by a motor. Each operation takes 8 seconds and five operations are done per minute. The machine is fitted with a flywheel having a mass of 200 kg with a mean radius of gyration of 400 mm. When the operation is being performed, the speed drops from the normal speed of 400 rpm to 250 rpm. Determine the power of the motor required. Also, find how much energy is used in performing each operation.

(4.28 kW, 51.3 kJ)

27. A shearing machine is used to cut flat strips and each operation requires 37.5 kN.m of energy. The machine has a flywheel with radius of gyration of 900 mm. The speed at the start of each operation is 1300 rpm. Determine the mass of the flywheel assuming that the energy required for cutting is fully supplied by the flywheel and the speed reduction is not more than 15% of the maximum. Also, find the torque supplied to the flywheel so that it regains its full speed in 3.3 seconds.

(1812.5 kg, 902.9 N.m)